Calculus I
Practice Problems 1: Answers

1. Find the equation of the line which goes through the point (2,-1) and is parallel to the line given by the equation $2x - y = 1$.

Answer. The given line has the equation $y = 2x - 1$; our line, being parallel will have equation $y = 2x + b$ for some b. Substitute $x = 2,\ y = -1$: $-1 = 2(2) + b$, so $b = -5$. The equation is $y = 2x - 5$.

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2. Find the equation of the line which goes through the point (2,-1) and is perpendicular to the line given by the equation $2x - y = 1$.

Answer. The given equation: $y = 2x - 1$ has slope 2. Thus the line we seek has slope -1/2. Its equation is

$$\frac{y-(-1)}{x-2} = -\frac{1}{2} \qquad or \qquad y = -\frac{1}{2}x\ .$$

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3. Find the equation of the line which goes through the point (1,2) and is parallel to the line through the points (0,1) and (-2,7).

Answer. The line through the points (0,1) and (-2,7) has slope

$$\frac{7-1}{-2-0} = -3\ .$$

So, our line through $(1,2)$ and slope $m = -3$ has the equation

$$\frac{y-2}{x-1} = -3$$

$$y - 2 = -3(x-1) = -3x + 3$$

$$y = -3x + 5$$

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4. Find the derivative: $f(x) = x^3 - x^2 + 1$

Answer. $3x^2 - 2x$.

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5. Find the derivative: $f(x) = x^5 + 3x^4 - 2x^2 + 4x - 7$

Answer. $5x^4 + 12x^3 - 4x + 4$.

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6. Find the derivative: $f(x) = x^4 - 2x^3 + 5x^2 - x + 7$

Answer. $4x^3 - 6x^2 + 10x - 1$.

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7. Find the derivative: $f(x) = 3x^{-1} + x^3$

Answer. $-3x^{-2}+3x^2$.

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8. Find the equation of the tangent line to the graph of $y = x^3 - 3x^2 + x$ at the point (2,-2).

Answer. Differentiating, we find $dy/dx = 3x^2 - 6x + 1$. Evaluate at $x = 2:$ $dy/dx = 3(2^2) - 6(2) + 1 = 1$. This is the slope of the tangent line at (2,-2), so its equation is

$$\frac{y-(-2)}{x-2} = 1 \quad or \quad y = x - 4\ .$$

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9. Let $y = 16x^{-1} - x^2$. At what point(s) is the tangent line horizontal?

Answer. Differentiating, $dy/dx = -16x^{-2} - 2x$. The tangent line is horizontal when its slope is zero. So, we solve

$$\frac{-16}{x^2} - 2x = 0 \quad or \quad \frac{-16-2x^3}{x^2} = 0\ ,$$

which has the solution $x = -2$. For this value of x, $y = 16(-2)^{-1} - 2^2 = -8 - 4 = -12$. The answer is (-2,-12).

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10. Let $y = 4x^4 + x$. At what point is the tangent line to the graph perpendicular to the line tangent to the graph at (0,0)?

Answer. Differentiating, $dy/dx = 16x^3 + 1$. At $x = 0$, we get $dy/dx = 1$; this is the slope of the tangent line at (0,0). A line perpendicular to this line has slope -1, so we must solve $dy/dx = -1$, or

$$16x^3 + 1 = -1\ , \qquad 16x^3 = -2\ ,$$

which has the solution $x = -1/2$. For this value of x, $y = 4(-1/2)^4 + 1 = 5/4$. Thus the point is (-1/2,5/4).

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11. Find the derivative: $f(x) = \left(x^2 + \frac{1}{x^3}\right)\left(x^3 - x^2 + 1\right)$

Answer. $\left(x^2 + x^{-3}\right)\left(3x^2 - 2x\right) + \left(2x - 3x^{-4}\right)\left(x^3 - x^2 + 1\right) = 5x^4 - 4x^3 + 2x + x^{-2} - 3x^{-4}$.

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12. Find f' and f'': $f(x) = \left(x + \frac{1}{x}\right)\left(x^2 + 1\right)$

Answer. Use the product rule:

$$f'(x) = (x + x^{-1})(2x) + (1 - x^{-2})(x^2 + 1)$$

$$= 2x^2 + 2 + x^2 + 1 - 1 - x^{-2} = 3x^2 - x^{-2} + 2\ .$$

Differentiating again,

$$f''(x) = 6x + 2x^{-3}\ .$$