Calculus I
Practice Problems 5

1. A curve is given by the equation $x^2 - xy + y^2 = 7$. Find the equation of the line tangent to this curve at the point (2,-1).

2. Find the slope of the curve defined by the relation $4(x^2 + xy) = 2y^3 - y^2$ at the point $(1,2)$.

3. Variables x and y are related by the formula $x \sin y + y \sin x = \pi$. If $dy/dt = 3$ when $x = 3\pi/2$ and $y = \pi/2$, what is dx/dt?

4. The relation $\cos y + x = \sin y$ determines a curve in the x-y plane. Find the slope of the line tangent to the curve at the point $(1, \pi/2)$.

5. Consider the curve given by the equation: $y^2 + xy + x^2 = 1$. At what points does this curve have a horizontal tangent line?

6. Consider the curve given by the equation: $x^2 y - y^3 = 1$. At what points does this curve have a vertical tangent line?

7. A ship is traveling in a circle of radius 6 nautical miles around an island at a speed of 10 knots. A lighthouse is 10 miles due east of the island. At what rate is the distance between ship and lighthouse increasing when the ship is exactly due north of the island?

8. A new stadium, built like a cylinder capped with a hemispherical dome is proposed to have a diameter of 500 feet. To include another 2000 seats, the diameter must be increased by 10 feet. By approximately how much will the area of the dome be increased? (Note: the area of a sphere of radius r is $4\pi r^2$.)

9. A cat is walking toward a telephone pole of height 30 feet. She is walking at a steady rate of 4 ft/sec. A bird is perched on top of the telephone pole. When the cat is 45 feet from the base of the pole, at what rate is the distance between bird and cat decreasing?

10. Water is flowing into a conical tank through an opening at the vertex at the top at the rate of 12 cu. ft./min. The base of the tank is a circle of radius 12 ft. and the height of the cone is 20 ft. At what rate is the water level rising when the water level is 4 ft. from the top? The formula for the volume of a cone of base radius r and height h is $V = (1/3)\pi r^2 h$.