Poisson random graphs (a.k.a "Erdos-Renyi random graphs", "random graphs", or "G(n,p)")

Random graphs are a convenient way to investigate the role of network structure e.g. in epidemics, communication networks, brain networks, etc. The most common random graph model is a Poisson random graph.

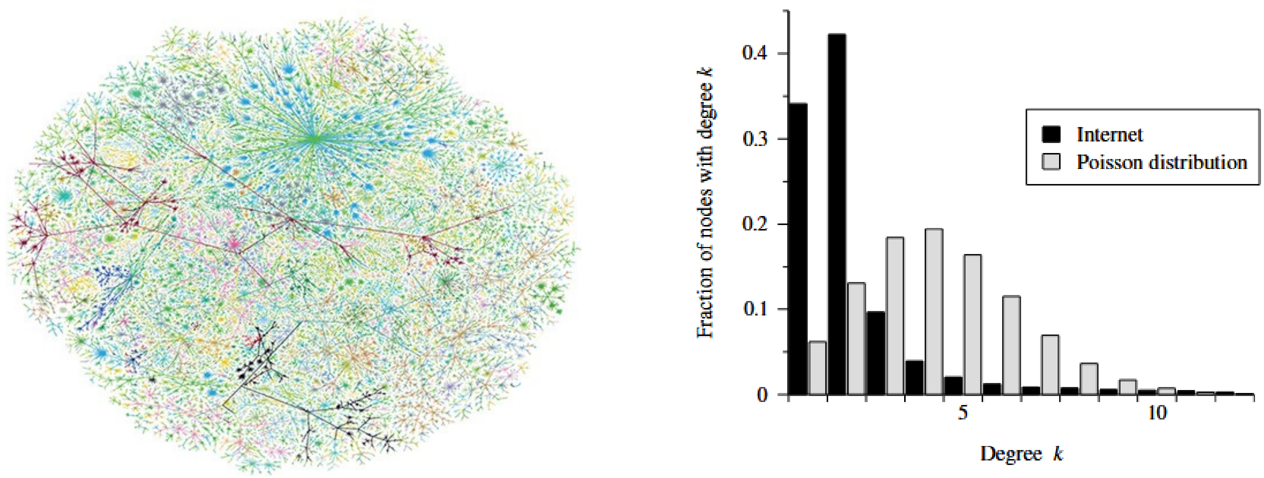
In a **Poisson random graph** an edge exists between any pair of distinct nodes with fixed probability

- Pro: relatively easy to construct
- Pro: nice properties
- Con: unrealistic

Poisson random graphs have in particular an unrealistic degree distribution:

- The **degree of a node** = number of edges involving the node
- Most real-world networks have "hubs" with a very high degree
- The hubs e.g. make real-world networks less susceptible to random attacks

(a) **(b)**



The network structure of the internet. **(a)** A visualization of the network structure of the internet at the level of autonomous systems. An autonomous system is a set of routers representing a single administrative entity such as a large university or an internet service provider. There were roughly 30,000 autonomous systems when the image was made in 2009. **(b)** The degree distribution of the internet is heavily skewed to the right, unlike Poisson distributions (as generated by Poisson random graphs).

1. Make a Poisson random graph $G(n,p)$ with $n = 100$ nodes and edge probability $p = 0.1$
2. Visualize the random graph using a computer
3. Investigate the degree distribution by finding each of the below (i) analytically and (ii) with your example network
   (a) Find the mean degree, $\langle k \rangle$. *Hint: the degree of a node is equal to the number of edge "stubs" on the node, so if the network has m edges, there are $2m$ edge stubs across the n nodes, making the average degree $\langle 2m/n \rangle$.*
   (b) Find the probability p_k that any particular node has degree k
   (c) Show that $p_k \approx e^{-\langle k \rangle} \frac{\langle k \rangle^k}{k!}$ assuming that the network is "sparse", i.e $\langle k \rangle << n$. *Hint: investigate this with your code by setting n to some large number and decreasing p so the average degree $\langle k \rangle$ is constant*
4. What might be some other problems with Poisson random graphs? *Hint: Do any features (or lack of features) of the graph you made seem a bit off?*