

Modeling and analysis of polymer gel dynamics

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Abstract

The present work is motivated by the swelling of body implantable biomedical devices. Devices made of polymer become gels once they are implanted and start absorbing fluid. We formulate a governing system of equations based on the theory of mixtures. Equations of balance of mass and linear momentum for the fluid and polymer components are obtained. The system couples equations of nonlinear elasticity with friction with the Navier-Stokes equations of fluid flow. We study the governing system in relevant regimes. Boundary conditions involve traction-displacement conditions for the elasticity problem together with statements about the permeability of the device interface for the fluid. The one-dimensional free boundary problem governing the swelling of the gel body becomes a fixed boundary problem in the Lagrangian formulation. We also study initial-boundary value problems for the coupled linear elasticity and Navier-Stokes equations in three-dimensional geometries. The latter reduces to the stress-diffusion coupling model of Doi when dissipation and change in volume are neglected. Difficulties with the governing system stem from the coexistence of the Eulerian and Lagrangian formulations, and also from the hyperbolic nature of the balance of mass equations. To deal with the latter, we regularize the mass equations in a thermodynamically consistent fashion by allowing for mass diffusion effects in the model. Since many gel problems are governed by parabolic or mixed parabolic/elliptic equations, a new DG method for solving parabolic equations is devised. This method is tested using the heat equation in one dimension as a model problem and rates of convergence are shown for the Dirichlet problem.