Ramsey numbers

• K_N is the *complete graph on n vertices.* It is defined as a collection of points $\{x_1, x_2, ..., x_N\}$ with an edge labeled $\overline{x_i x_j}$ for each distinct pair of vertices x_i and x_j.

• A red/blue coloring of K_N is a choice of color — either red or blue — made for each edge in K_N.

• A red/blue coloring of K_N *contains a red K_m* if there are m vertices of K_N such that each edge connecting pairs of the distinguished m vertices is red.

• K_N has property (m, n) if any red/blue coloring of K_N contains either a red K_m or a blue K_n.

• $R(m, n)$ is the smallest of all the numbers N with K_N having property (m, n).

• For arbitrary numbers m and n, it is not clear that there should be any number N as in the previous sentence, let alone a smallest one.

• **Goal:** Given numbers m and n, find $R(m, n)$.

- $R(m,n) = R(n,m)$

- $R(m,n) \leq R(m-1,n) + R(m,n-1)$

- $R(m,n) \leq \binom{m+n-2}{m-1}$

- For $k \geq 4$, $2^{\frac{k}{2}} \leq R(k,k) \leq 2^{2k-3}$

- All known Ramsey numbers: $R(2,k) = k$; $R(3,3) = 6$; $R(3,4) = 9$; $R(3,5) = 14$; $R(3,6) = 18$; $R(3,7) = 23$; $R(3,8) = 28$; $R(3,9) = 36$; $R(4,4) = 18$; and $R(4,5) = 25$.

- $43 \leq R(5,5) \leq 52$

- $102 \leq R(6,6) \leq 169$

- Paul Erdos on finding Ramsey numbers:

Suppose an evil alien would tell mankind "Either you tell me [the value of $R(5,5)$] or I will exterminate the human race". ... It would be best in this case to try to compute it, both by mathematics and with a computer.

If he would ask [for the value of $R(6,6)$], the best thing would be to destroy him before he destroys us, because we couldn't [determine $R(6,6)$].