Classical Problems in Commutative Algebra, Week 2

-speakers & tentative lecture plans-

Sankar Dutta (Univ. of Illinois, Urbana-Champaign): 4 lectures
Intersection Multiplicity, Chow Groups and Canonical Element Conjecture

1. Intersection Multiplicity and Higher Euler Characteristics: some old and some new results.

2. Continuation of No.1 if required and the Chow group problem.

3. Smoothness - a link among Bass-Quillen Conjecture, Intersection Multiplicity and the Chow groups.

4. Canonical Element Conjecture, Local Cohomology and a Characteristic p (positive) approach.

Ray Heitmann (University of Texas): 4 lectures
Rigidity, Closure Operations and the Direct Summand Conjecture

1. Introduction to rigidity, Auslander and Lichtenbaum results and the counterexample.

2. What we want in a closure operation and a careful study of plus closure, including the Hochster-Huneke result that R+ is C-M in equicharacteristic p. This may be too ambitious for a single talk and likely will run over into the third lecture.

3. Introduce extended plus closure and the theorem that colon-capturing implies ideals in regular rings are closed.

4. The direct summand conjecture.

Melvin Hochster (University of Michigan): 5 lectures
Local Homological Conjectures, Big Cohen-Macaulay Algebras, and Tight Closure

Some historical perspectives on the local homological conjectures, and implications among them (to the extent that other speakers have not done them)

A somewhat detailed treatment of the canonical element conjecture and how it can be used to prove other local homological conjectures, including the syzygy conjecture, as well as why it follows from the direct summand conjecture.

Applications of the weakly functorial existence of big Cohen-Macaulay algebras, as well as why they exist and the known cases (equal characteristic, pairs of rings of mixed characteristic of dimension at most three). In particular, the vanishing conjecture for maps of Tor.

In-depth treatment of the vanishing conjecture for maps of Tor, including its equivalence, following Nandini Ranganathan, with a splitting conjecture that generalizes the direct summand conjecture. Explanation of how it implies that direct summands of regular rings are Cohen-Macaulay and the direct summand conjecture.

Connections between tight closure theory and the local homological conjectures. Tight closure proof of the vanishing conjecture for maps of Tor.
Approaches to generalizing tight closure theory to mixed characteristic.

Paul Roberts (University of Utah): 5 lectures
Intersection properties of modules of finite projective dimension

1. Intersection Theorems. I plan to include a proof of why the Peskine-Szpiro Intersection Theorem implies the Auslander and Bass Conjectures as well as a proof of the New Intersection Theorem. I will mention the improved version.

2. Local Chern Characters--what they are and what they are good for.

3. Gabber's proof of the Serre nonnegativity conjecture.

4. Modules of finite length and finite projective dimension.

5. Annihilators of local cohomology of small valuation.