Proposal for REU Fall 2009

Proposal title: Estimation of the distribution of spontaneous emissions from quantum dots with the inverse Laplace transform
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1 Background

In many applications such as transistors, solar cells, LEDs, and diode lasers it is important to control the spontaneous photon emissions from atoms. Photonic crystals are known to affect the spontaneous emissions. One way of quantifying this effect is to embed quantum dots in the photonic crystal. An isolated quantum dot absorbs light at a given frequency and emits it with an exponentially decaying intensity with known decay rate. However, when many quantum dots are embedded in a photonic crystal the emission intensity $I(t)$ at time t can be modeled by,

$$I(t) = I(0) \int_{\gamma=0}^{\infty} \phi(\gamma) \exp[-\gamma t] d\gamma, \tag{1}$$

where $\phi(\gamma)$ is the distribution of concentration of emitters for a certain decay rate γ. We notice that in the physical model the intensity is essentially the Laplace transform of the distribution, i.e. $I(t)/I(0) = \mathcal{L}[\phi(\gamma)](t)$. According to Vos et al. [2] a good model for $\phi(\gamma)$ is the log-normal distribution:

$$\phi(\gamma) = A \exp\left[-\frac{\ln^2(\gamma/\gamma_{MF})}{w^2} \right], \tag{2}$$

where A is a normalization constant such that $\int_0^\infty \phi(\gamma) d\gamma = 1$.

2 Mathematical Problem

The problem of determining $\phi(\gamma)$ from its Laplace transform $I(t)$ is known to be a severely ill-posed linear inverse problem, which means that small perturbations in $I(t)$ can lead to large errors in the estimation of $\phi(\gamma)$. By using the empirical model (2) the problem is reduced to estimating two unknown parameters γ_{MF} and w. This is one way of regularizing the problem. I propose to study other regularization methods for the inverse Laplace transform such as the one by

Epstein and Schotland [1]. Also the study of the inverse Laplace transform [1] could be used to quantify the uncertainty in the estimation due to the presence of noise in the data and give estimates of the minimum time sampling.

3 Objectives

- Understand common techniques for linear inverse problems.
- Apply common regularization techniques to the inverse Laplace transform problem.
- Study whether the choice (2) for $\phi(\gamma)$ is a good choice from the prospective of inverse Laplace transform.
- Use the results in [1] to determine the uncertainty in estimation, practical limits to what can be estimated, and what time sampling of the intensity is sufficient, given the accuracy of the measurements.

References

[1] Charles L. Epstein and John Schotland. The bad truth about Laplace's transform. *SIAM Rev.*, 50(3):504–520, 2008.

[2] Ivan S. Nikolaev, Peter Lodahl, A. Floris van Driel, A. Femius Koenderink, and Willem L. Vos. Strongly nonexponential time-resolved fluorescence of quantum-dot ensembles in three-dimensional photonic crystals. *Physical Review B (Condensed Matter and Materials Physics)*, 75(11):115302, 2007.