

## Fall 2007 VIGRE Report

With relatively recent discoveries in mathematical approaches to pricing financial instruments, much research has been performed seeking to create more accurate models to be used for investment purposes. One of the areas studied is how to price debt, often in the form of bonds, which companies sell in attempts to raise capital with the promise of interest payments of some sort. This investment involves risk due to the chance that a company may go bankrupt and pay back less than the total owed debt, or possibly none at all. There are indicators which may be used to measure a company's riskiness, also called credit ratings, which are independently created by organizations such as Moody's or Standard and Poor's. They measure the quality of the debt issued by companies, particularly the likelihood of default for a company. This helps investors decide how much should be paid for a given bond. However, an impediment to analyzing the credit risk is the variable nature of firms' credit ratings, which fluctuate with time. The ratings can change as the company does or does not have the earnings they were expecting for that year, or as consumer confidence changes. Theories of mathematical finance may be used to price derivatives on financial securities, thereby giving investors information on the riskiness of an investment. Some of the means by which the prices of these derivatives is determined will be discussed, along with areas where some improvements may be made.

Theories for pricing financial derivatives are subject to the accuracy of models used which describe the stochastic nature of credit risk. Many of the models which I have studied use the probability of default for a company to price the time zero value of a bond sold by this company. For example, Jarrow and Turnbull derive bond-pricing formulas which use calculated pseudoprobabilities of default given by the initial term structure of debt.[1] Its methods are based on the chance a company will default, and also gives values for recovery rates on the investment, the percentage

salvaged if a company does go bankrupt. Though the calculated pseudoprobabilities of default in this paper are sufficient to make the market complete with no arbitrage opportunities, this is based only on the initial term structure. Over a larger time horizon, the probability of default is difficult to estimate with a great amount of accuracy. Other methods base the probability of default on past observations,[2] but it is still difficult to accurately depict what actually occurs. This is because a firm with a high rating may slowly default, moving from higher credit classes to lower credit classes until it defaults. Other firms may swiftly decline, skipping credit classes. These aggregated observations are taken into account for future predictions. This means that anomalous behavior may have been previously exhibited, further impairing efforts to accurately model this default probability. Because of this, a lot of research is being done to improve modeling of which default probability to prescribe.

This problem can be simplified by using a Markov chain process for the probability of change in credit rating for a given firm. It is assumed that the Markov assumption holds, that the previous credit rating is the only relevant information for determining the next state of a company's credit rating. The reasonableness of this assumption is debatable for several reasons. One of these reasons deals with momentum effects. This is when the chance of a downgrade is increased by a previous downgrade, or the chance of an upgrade is decreased by a previous downgrade, as studied in [3, pp. 97-102]. Disregarding these momentum effects, a Markov chain can be produced from a generator matrix created by collecting data on firms. The elements of this matrix may be computed by approximating the probability of jumping from one credit class to another by past movements. The probability of moving from credit class  $A$  to  $B$  is equal to the number of firms which moved from  $A$  to  $B$  divided by the total time spent by firms in  $A$ . This is reasonable, since we can only base future observations on what has happened in the past. Repeating this process for every combination of firm movements gives a generator matrix,  $\Lambda$ . The  $ij$ th entry of the matrix is given

as the observed likelihood for a company moving from the  $i$ th credit class to the  $j$ th credit class. The maximum likelihood estimator for a time horizon  $t$  is  $P(t)$ , which may be approximated using a Taylor's expansion series, [3]

$$P(t) = e^{t\Lambda} = \sum_{k=0}^{\infty} \frac{(t\Lambda)^k}{k!}, t \geq 0.$$

Deriving  $P(t)$  from the generator matrix is useful because it gives non-zero probabilities to some unlikely events. For example, it may give a probability of .005 for the chance of default from the highest credit class to default in one step, despite there being no observed transitions of this sort in the data. The last column of this matrix will give the probabilities for default, which is used by pricing methods as described earlier. The bottom row is filled with 0's and a terminating 1 in the lower-right entry, meaning that there is no chance for a company to return to a non-default rating after defaulting.

The purpose of my current research, funded by a VIGRE Grant and supervised by Professor Jingyi Zhu, is to create a more accurate pricing method which allows for seasonal periodic fluctuations in credit ratings. This would use the same sort of pricing model as [1] or [2], but it would seek to more accurately depict fluctuations which occur in the probabilities of default and changes in credit classes. This would be a better description because the current model assumes a generator matrix which is the same from year to year. This does not make intuitive sense, since markets are constantly fluctuating, and more information may become available at a later date. Credit rating models should reflect this phenomenon, and some efforts have been made to study it by using a nonhomogeneous Markov chain as in [3, pp. 150-152].

What I have worked on so far is using a matrix  $\Lambda(t)$  which consists of the standard generator matrix  $\Lambda$  created from analysis of Standard and Poor's bond price information and another matrix,

$\Lambda_0(t)$ . The first matrix is created using the spread, the difference between the Treasury Bill's interest rate and the rates given by firms of different credit classes. This defines the stationary portion of the matrix,  $\Lambda$ , and then the periodic term is introduced. This is using a matrix  $\Lambda_0(t)$ , which is multiplied by  $\epsilon$  and  $\cos(\omega t)$ ,  $\epsilon$  and  $\omega$  constants, to control its influence and make it periodic. This matrix has not been determined as of yet, so a simple symmetric matrix which follows the basic rules of a generator matrix is used. This makes it so that adding the two generator matrices  $\Lambda$  and  $\Lambda_0(t)$  creates a new generator matrix. What needs to be done is research what form the  $\Lambda_0(t)$  matrix should take, and then adjust the values to make it better approximate the actual values. So far, this method has produced results that are similar to what is desired. Basing the values off of the spread alone seem to eliminate certain forms of market behavior, which generally tend to inflate the price of bonds, so this may be more accurate in a sense. However, a lot of further research must go into it to be of any use.

This problem has many facets, and is interesting due to the number of factors which affect the answer. Since the market for things such as bonds is constantly changing, it is difficult to really claim that a great model can be produced. However, research into creating a more-improved model is warranted as it is very important for determining prices of certain financial instruments. It also helps to portray the status of different firms, and how that can change over time.

## References

- [1] Jarrow, R., and S. Turnbull, 1995, "Pricing Derivatives on Financial Securities Subject to Credit Risk," *The Journal of Finance*, 50, 53-85.
- [2] Jarrow, R., D. Lando, and S. Turnbull, 1997, "A Markov Model for the Term Structure of Credit Risk Spreads," *The Review of Financial Studies*, 10, 481-523.

- [3] Lando, D., 2004, *Credit Risk Modeling: Theory and Applications*, Princeton University Press, New Jersey.