$h - \varepsilon$ Refinement of Finite Difference Formulas Generated by Radial Basis Functions

Student: Eric Heisler, Mentor: Grady Wright

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1 Introduction

Radial basis function generated finite difference formulas (RBF-FD) were recently introduced as a novel technique for generalizing finite difference formulas to solve PDEs on irregular geometries. While the RBF-FD method has proven useful for applications ranging from fluid dynamics to structural mechanics, little is known about its theoretical properties, i.e. error bounds and convergence. Central to studying these properties is understanding the role of the shape parameter, ε, inherent to RBF approximations.

It has widely been observed that the value of the shape parameter greatly affects the accuracy of the method. With the optimal choice, the accuracy can be orders of magnitude better than standard finite difference formulas. During spring 2007, we constructed an algorithm for computing the optimal choice of ε for several common differential equations. The results of many numerical tests have shown that this algorithm for ε refinement can give far lower error as well as a higher order of accuracy compared to the standard method. Many types of differential equations and conditions remain to be examined as well as some theoretical concerns which have not been addressed.

2 Project Goals

This project is a continuation of the spring 2007 REU project. There are two primary sets of goals which we hope to accomplish.

- First, we will develop and test this method on a broader range of applications. There are several differential equations and conditions which we would like to apply this method to such as a wave equation in one and two dimensions, a non-linear elliptic PDE and certain problematic conditions encountered during the spring.

- Second, there are several situations where the behavior of the method is not well known. It is necessary to examine these cases from a theoretical standpoint as well as perform numerical tests to determine the effectiveness of the method for a more general set of problems.

The method has been developed and tested for poisson's equation on a unit square, a poisson equation on a square with re-entrant corners, and with the multigrid method. The problems to be examined for his project include a wave equation in one and two dimensions, a non-linear elliptic PDE and convection-diffusion equation. All of the computational work will make use of Matlab.