Math 6210 - Homework 2

Due in class on 9/15/10

From Rudin: Chapter 1, # 3,5,6,11,12

Let $f : [0, 1] \to [0, \infty)$ be a bounded, Riemann integrable function. Show that f is measurable and that the Riemann integral and Lebesgue integral agree. Here is one outline of a proof.

1. Find simple measurable functions, f_i^+ and f_i^-, such that $f_i^+ \geq f_{i+1}^+ \geq f$, $f_i^- \leq f_{i+1}^- \leq f$ and in both cases the limit of the Riemann and Lebesgue integrals of the sequences converges to the Riemann integral of f.
2. Let $h : [0, 1] \to [0, \infty)$ be a measurable function such that $h^{-1}((0, \infty))$ has positive measure. Show that the Lebesgue integral of h is positive.
3. Let $f^+ = \underset{i \to \infty}{f_i^+}$ and $f^- = \underset{i \to \infty}{f_i^-}$. Use (b) to show that the set of points where $f^+ \neq f^-$ has measure zero. Conclude that $f = f^-$ (or f^+) outside of a set of measure zero.
4. Let f_0 and f_1 be functions such that $f_0 = f_1$ a.e. Show that if f_0 is measurable then f_1 is measurable.
5. Conclude that f is measurable and that the Riemann and Lebesgue integrals agree.