Math 6510 - Homework 2

Due in class on 9/19/13

1. Assume that M and N are submanifolds of Euclidean space and that $f : M \to N$. Show that f determines a diffeomorphism between TM and TN.

2. Recall that $M(n)$ is the space of $n \times n$ matrices and is naturally identified with $\mathbb{R}^{n^2}$. Let $SL(n) = \{A \in M(n) | \det A = 1\}$. Show that $SL(n)$ is a differentiable submanifold and show that the tangent space at the identity is the subspace of all matrices of trace zero.

3. Let $M = \{(x_0, x_1, x_2, x_3) \in \mathbb{R}^4 | x_0^2 + x_1^2 = x_2^2 + x_3^2 = 1\}$. Show that M is a differentiable submanifold of $\mathbb{R}^n$. Given an explicit description of TM and show that it is diffeomorphic to $M \times \mathbb{R}^2$. Can you give another description of this manifold?