Math 6510 - Homework 8

Due in class on 12/12/13

1. Let $T \in \mathcal{T}^k(V)$ and $S \in \mathcal{T}^l(V)$ be tensors on a vector space V with $Alt(S) = 0$. Show that $Alt(T \bigotimes S) = Alt(S \bigotimes T) = 0$.

2. Let $\bar{\omega} : \Lambda(M) \times \cdots \times \Lambda(M) \to C^\infty(M)$ be a function such that $\bar{\omega}(X_1, \dots, fX_i + gX_i', \dots, X_k) = f\bar{\omega}(X_1, \dots, X_i, \dots, X_k) + g\bar{\omega}(X_1, \dots, X_i', \dots, X_k)$. Show that there exists a $\omega \in \mathcal{T}^k(M)$ with $\omega = \bar{\omega}$.

3. Let α be a k-form and show that $d(\alpha \wedge \beta) = d\alpha \wedge \beta + (-1)^k \alpha \wedge d\beta$. This can be found in many books but you should try to do it yourself.

4. Let $dx_I = dx_{i_1} \wedge \cdots \wedge d_{i_k}$ and $dx_{I'} = dx_{i_1} \wedge \cdots \wedge d_{i_{k-1}}$. Let $\omega = g dx_I$ and $\omega' = g dx_{I'}$. Show that $d\omega = d\omega' \wedge dx_{i_k}$.

5. Let M be an n-dimensional manifold. Show that the bundle of n-forms is a product if and only if M is orientable.

6. For the product manifold $M \times I$, where I is and interval, let $\pi_M : M \times I \to M$ and $\pi_I : M \times I \to I$ be the projections to each factor. If ω is a k-form on $M \times I$ show that there exists one parameter families of k-forms α_t and $k-1$-forms η_t, both on M, such that

$$\omega(p,t) = (\pi_M^* \alpha_t)(p,t) + (\pi_I^* dt \wedge \pi_M^* \eta_t)(p,t).$$