Introductory topics in Kleinian grops and hyperbolic 3-manifolds
Chuckrow's theorem

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The following theorem was supposed to have been proven in lecture 2. It wasn't so now it becomes homework!

Theorem 0.1 (Chuckrow, Jorgensen) *Let ρ_n be a sequence of discrete faithful representations of a torsion free group G in* $\mathrm{Isom}^+(\mathbb{H}^3)$ *that converges to a representation ρ. If G is not abelian then ρ is discrete and faithful.*

A *discrete, faithful representation* of G is an injective homomorphism from G to $\mathrm{Isom}^+(\mathbb{H}^3)$ where the image is discrete. We say $\rho_n \to \rho$ if for all $g \in G$, $\rho_n(g) \to \rho(g)$ in $\mathrm{Isom}(\mathbb{H}^3)$.

Here is one way to prove this.

1. If ρ is not discrete show that (after possibly passing to subsequence) that there exists $g_n \in G \backslash \{\mathrm{id}\}$ such that $\rho_n(g_n) \to \mathrm{id}$.

2. Observe if ρ is not faithful that there exists a $g \in G \backslash \{\mathrm{id}\}$ such that $\rho_n(g) \to \rho(g) = \mathrm{id}$. In the remaining exercises we take $g_n = g$ to be a constant sequence when ρ is not faithful.

3. Let $h \in G$ be an arbitrary element and show that $\rho_n([h, g_n]) \to \mathrm{id}$.

4. Given any $p \in \mathbb{H}^3$ show that for large n both $\rho(g_n)$ and $\rho_n([h, g_n])$ translate p some distance $< \epsilon_3$ where ϵ_3 is the 3-dimensional Margulis constant.

5. For large n show that $[h, g_n]$ and g_n commute.

6. If $a, b \in \mathrm{Isom}^+(\mathbb{H}^3)$ show that if a and $[a, b]$ commute then a commutes with b. Use this to show that h and g_n commute for large n.

7. Show that if $a, b, c \in \mathrm{Isom}^+(\mathbb{H}^3)$ and a commutes with both b and c then b commutes with c. Use this to show that G is abelian. The proof is done!

8. We've actually shown something a bit stronger. Let $\Gamma_n = \rho_n(G)$ be the ρ_n-image of G. If G is not abelian and ρ_n converges then the identity is isolated in the union $\bigcup_n \Gamma_n$. Why?