Assume that L is a Dedekind cut. We'll show that

$$K = \{ x \in \mathbb{Q} \mid \exists y \notin L \text{ with } x+y<0 \}$$

is a Dedekind Cut.

Since L is a Ded. cut, $L \neq \mathbb{Q}$ & $\exists y \notin L$. Then $-y-1 \in \mathbb{Q}$ & $(-y-1)+y = -1 < 0$ which implies that $-y-1 \in K$ & $K \neq \emptyset$.

Fix $z \in L$, Then $x = -z+1$ is in $\mathbb{Q}$ & $x+z = 1 > 0$. For all $y \notin L$, $y > 0$ so $x+y > x+z > 0$. Therefore $x \notin K$ & $K \neq \mathbb{Q}$.

If $x \in K$ $\exists y \notin L$ s.t. $x+y<0$. By the Archimedean prop $\exists n \in \mathbb{N}$ s.t. $0 < \frac{1}{n} < -(x+y)$. Then $x' = x + 1/n$ & $x'+y = x + \frac{1}{n} + y < 0$ so $x' > x$ & $x' \in K$. Therefore K has no largest element.

If $x \in K$ $\exists$ $y \notin L$ s.t. $x+y<0$. If $x' < x$ then $x'+y < x+y < 0$ so $x' \in K$. Therefore if $x' < x$ & $x \in K$ then $x' \in K$.

K therefore satisfies the 3 properties of a Dedekind Cut.