Hyperbolic geometry in dimensions 2 and 3

This course will be an introduction to some topics in hyperbolic geometry. Our main focus will be surfaces although we will also spend some time discussing hyperbolic 3-manifolds later in the semester. The class will be accessible to anyone who has taken the first year graduate courses. If you haven't completed all of the first year courses it is possible that you will still get something out of this course but you should speak with me first. The course should be a good introduction to some important topics in the intersection of geometry, topology and analysis.

Below is a list of some of the possible topics that we will cover. I will also be running the Stallings seminar (student geometry/topology seminar) this semester. There will be many places were I will state theorems without proof and these will be good topics for Stallings seminar presentations.

- Conformal metrics in the plane
- Models for the hyperbolic metric
- Hyperbolic trigonometry
- $PSL_2\mathbb{R}$ and $SO(2,1)$ as isometries of the hyperbolic plane
- Hyperbolic surfaces
- Markings
- Teichmüller space
- Fenchel-Nielsen coordinates
- Marked hyperbolic surfaces as representations
- quasi-conformal maps - geometric and analytic definitions
- extremal length
- Beltrami and quadratic differentials
- complex line bundles
- the measurable Riemann mapping theorem (without proof)
- Grötszch's theorem - Teichmüller's theorem
- Hyperbolic 3-space

- $PSL_2\mathbb{C}$ and $SO(3,1)$ as isometries of hyperbolic 3-space
- Basic Kleinian group definitions - limit set, domain of discontinuity, convex hull, convex core, geometric finiteness
- quasi-isometries and there extensions to quasi-conformal maps at infinity
- rigidity theorems and deformations theorems
- the Schwarzian derivative
- the Bers' embedding and Teichmüller space as a complex manifold
- The Weil-Petersson metric - definition and basic facts
- Epstein surfaces
- Renormalized volume