$p : (E, e_0) \to (B, b_0)$

$e_1 \in p^{-1}(b_0)$

Show that $\exists\ p_1 : E \to E$ s.t.

$p_1(e_1) = e_0$ & $p \circ p_1 = p$.

1

$$\begin{tikzcd} & (E, e_0) \arrow[d, "p"] \\ (E, e_1) \arrow[ur, "p_1"] \arrow[r, "p"] & (B, b_0) \end{tikzcd}$$

2

$$\begin{tikzcd} & (E, e_1) \arrow[d, "p"] \\ (E, e_0) \arrow[ur, "p_1"] \arrow[r, "p"] & (B, b_0) \end{tikzcd}$$

Want to show $p_0 \circ p_1 = p_1 \circ p_0 = id$

$$\begin{tikzcd} & (E, e_0) \arrow[d, "p"] \\ (E_0, e_0) \arrow[ur, "\tilde{p}"] \arrow[r, "p"] & (B, b_0) \end{tikzcd}$$

$\underline{id}$ is such a lift as is $\underline{p_0 \circ p_1}$

$\boxed{\text{LEMMA}}$ Let $q : E \to E$ s.t. $p \circ q = p$ & $q(x) = x$ for some $x \in E$

$\Rightarrow q = id.$

$\boxed{\text{PROOF}}$ Apply Fnd lifting lemma ~~to~~

$$\begin{tikzcd} & (E, x) \arrow[d, "p"] \\ (E, x) \arrow[ur] \arrow[r, "p"] & (B, p(x)) \end{tikzcd}$$

q & id are both lifts with $q(x)=x$
& $id(x)=x$

$\Rightarrow$ $q = id$. $\blacksquare$

COR Let $q_0, q_1 : E \to E$ s.t.
$p \circ q_i = p$ & $q_0(x) = q_1(x)$ for some $x \in E$.
$\Rightarrow$ $q_0 = q_1$.

PF

$$\begin{array}{ccc} & & (E, q_0(x)) \\ & \overset{q_0, q_1}{\nearrow} & \downarrow p \\ (E, x) & \xrightarrow{p} & (B, p(x)) \end{array} \quad \Rightarrow \quad q_0 = q_1$$

Apply previous to $q_0 \circ q_1^{-1}$. $\blacksquare$

$$p^{-1}(b_0) \longrightarrow \text{homeo}(E)$$
$$e_i \longmapsto p_i$$

where $p_i(e_i) = e_0$ & $p \circ p_i = p$.

Show that the image is a subgroup G.

<u>comp</u> Need to show $p_1 \circ p_2 \in G$.

$$p \circ (p_1 \circ p_2) = (p \circ p_1) \circ p_2 = p \circ p_2 = p$$

Need to find $e_3 \in p^{-1}(b_0)$ s.t.

$p_1 \circ p_2(e_3) = e_0$ $\Rightarrow$ $p_1 \circ p_2 = p_3 \in G$

$e_3 = p_2^{-1}(e_1)$.

To use this need to know that $p_2^{-1} \in G$.

By (2) we know p_2^{-1} is a homeo.

Also let $e_{-2} = p_2(e_0)$. Then $\underset{\underset{e_0}{\shortparallel}}{p_{-2}(e_{-2})} = \underset{\underset{e_0}{\shortparallel}}{p_2^{-1}(e_{-2})}$

By lemma $p_{-2} = p_2^{-1} \Rightarrow p_2^{-1} \in G$.

(4) Show that $\underline{G}$ is a $\underline{\text{deck}}$ action.

$U \subset B$ is $\underline{\text{evenly}}$ $\underline{\text{covered}}$ if $p^{-1}(U) = \coprod V_\alpha$ where $p|_{V_\alpha}$ is a homeo to U.

Pick $x \in E$, assume U is an evenly covered nbd of $p(x)$ in B. Let V be the component of $p^{-1}(U)$ that contains x. $p|_V$ is injective.

Let $p_i \in G$. Show if $p_i(U) \cap V \neq \emptyset$ $\Rightarrow p_i = id$.

Let $y \in p_i(V) \cap V$. Then $p(p_i(y)) = p(y)$
$\Rightarrow p_i(y) = y$ since $p|_V$ is injective
$\Rightarrow p_i = id$.

$$\begin{tikzcd} & E \arrow[dl, "q"'] \arrow[dr, "p"] & \\ E/G \arrow[rr, squiggly, "f"] & & B \end{tikzcd}$$

To show f is well defined & a bijection need to show that

$$q(x) = q(y) \iff p(x) = p(y).$$

$\Rightarrow$ $\exists\, p_1 \in G$ with $p_1(x) = y$

$$\Rightarrow p(y) = p(p_1(x)) = (p \circ p_1)(x)$$
$$= p(x).$$

$\Leftarrow$ $p(x) = p(y)$, By the lifting lemma $\exists\, p_1 : E \to E$ with $p_1(x) = y$ & $p \circ p_1 = p$,

let $e_1 = p_1^{-1}(e_0)$ $\Rightarrow$ $p_1 \in G$.

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