

$$p: (E, e_a) \rightarrow (B, b_a)$$

$$e_a \in p^{-1}(b_a)$$

Show that $\exists p_1: E \rightarrow E$ s.t

$$p_1(e_a) = e_a \quad \& \quad p \circ p_1 = p.$$

$$1 \quad \begin{array}{ccc} & p_1 \nearrow & (E, e_a) \\ & & \downarrow p \\ (E, e_a) & \xrightarrow{p} & (B, b_a) \end{array}$$

$$2 \quad \begin{array}{ccc} & p_1 \nearrow & (E, e_a) \\ & & \downarrow p \\ (E, e_a) & \xrightarrow{p} & (B, b_a) \end{array}$$

Want to show $p_0 \circ p_1 = p_1 \circ p_0 = id$

$$\begin{array}{ccc} & \tilde{p} \nearrow & (E, e_a) \\ & & \downarrow p \\ (E_0, e_0) & \xrightarrow{p} & (B, b_0) \end{array} \left| \begin{array}{l} id \text{ is such a lift} \\ \text{as is } \underline{p_0 \circ p_1} \end{array} \right.$$

LEMMA

Let $q: E \rightarrow E$ s.t

$$p \circ q = p \quad \& \quad q(x) = x \text{ for some } x \in E$$

$$\Rightarrow q = id.$$

PROOF

Apply the lifting lemma to

$$\begin{array}{ccc} & \nearrow & (E, x) \\ & & \downarrow p \\ (E, x) & \xrightarrow{p} & (B, p(x)) \end{array}$$

g & id are both left with $g(x) = x$
& $id(x) = x$

$\Leftrightarrow g = id$. □

COR Let $g_0, g_1 : E \rightarrow E$ s.t
 $f \circ g_i = p$ & $g_0(x) = g_1(x)$ for some $x \in E$.
 $\Rightarrow g_0 = g_1$.

PA

$$\begin{array}{ccc} g_0, g_1 & (E, g_0(x)) & \\ \downarrow p & & \Rightarrow g_0 = g_1 \\ (E, x) & \xrightarrow{p} (b, p(x)) & \end{array}$$

Apply previous to $g_0 \circ g_1^{-1}$. □

$$\begin{array}{ccc} p^{-1}(s_0) & \rightarrow & \text{homeo}(E) \\ e_i & \mapsto & p_i \end{array}$$

where $p_i(e_i) = e_0$ & $p \circ p_i = p$.

Show that the image is a subgroup G .

comp Need to show $p_1 \circ p_2 \in G$.

$$p \circ (p_1 \circ p_2) = (p \circ p_1) \circ p_2 = p \circ p_2 = p$$

Need to find $e_3 \in p^{-1}(s_0)$ s.t

$$\begin{array}{ccc} p_1 \circ p_2(e_3) = e_0 & \Leftrightarrow & p_1 \circ p_2 = p_3 \in G \\ e_3 = p_2^{-1}(e_1) & & \end{array}$$

To use this need to know that $p_i^{-1} \in G$.
 By (2) we knew p_i^{-1} is a homeo.



Also let $e_{-2} = p_2(e_0)$. then $p_{-2}(e_{-2}) = p_2^{-1}(e_{-2})$
 $\quad \quad \quad \uparrow \quad \quad \quad \uparrow$
 $\quad \quad \quad e_0 \quad \quad \quad e_0$

By lemma $p_{-2} = p_2^{-1} \Rightarrow p_2^{-1} \in G$.

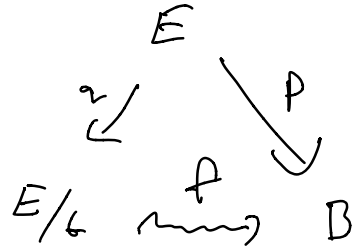
(4) Show that G is a deck action.

$U \subset B$ is evenly covered if
 $p^{-1}(U) = \bigsqcup V_\alpha$ where $p|_{V_\alpha}$ is
 a homeo to U .

Pick $x \in E$, assume U is an evenly covered nbhd of $p(x)$ in B . Let V be the component of $p^{-1}(U)$ that contains x .
 $p|_V$ is injective.

Let $p_i \in G$. Show if $p_i(V) \cap V \neq \emptyset$
 $\Rightarrow p_i = \text{id}$.

Let $y \in p_i(V) \cap V$. Then $p(p_i(y)) = p(y)$
 $\Rightarrow p_i(y) = y$ since $p|_V$ is injective
 $\Rightarrow p_i = \text{id}$.



To show f is well defined & a bijection
 need to show that

$$q(x) = q(y) \Leftrightarrow p(x) = p(y).$$

$$\begin{aligned}
 \Rightarrow \exists p_i \in G \text{ with } p_i(x) = y \\
 \Rightarrow p(y) = p(p_i(x)) = (p \circ p_i)(x) \\
 = p(x).
 \end{aligned}$$

$$\Leftarrow p(x) = p(y) \text{ , by the first lemma } \\
 \exists p_i : E \rightarrow E \text{ with } p_i(x) = y \text{ \& }$$

$$p \circ p_i = p.$$

$$\text{let } e_i = p_i^{-1}(e_0) \Rightarrow p_i \in G.$$

2 ')