MATH 2270

Quiz #2 - Fall 2008

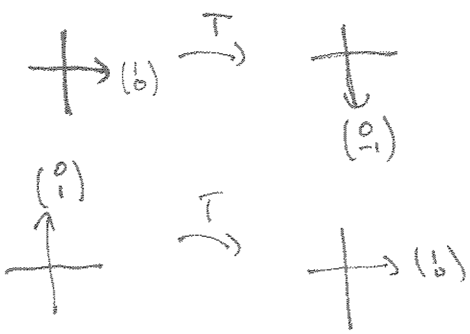
Name: Answer Key

1. (5 points) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation given by rotating a vector $\vec{x} \in \mathbb{R}^2$ *clockwise* by 90 degrees. Then T can be written as

$$T(\vec{x}) = A\vec{x}$$

for some matrix A.

(a) Compute the matrix A.



$$\therefore \quad A = \begin{pmatrix} | & | \\ T(\vec{e}_1) & T(\vec{e}_2) \\ | & | \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(b) Is T injective (i.e. 1-1)?

Yes

(c) Is T surjective (i.e. onto)?

Yes

(d) Is T invertible?

Yes

2. (2 points) Write the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ as a linear combination of the standard vectors $\vec{e}_1, \vec{e}_2, \vec{e}_3$.

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = 1 \cdot \vec{e}_1 + 2\vec{e}_2 + 3\vec{e}_3$$

3. (4 points) Compute the inverse of the following matrix using Gauss-Jordan elimination. Show your work.

$$\begin{pmatrix} 1 & 1 \\ 1 & 2 \end{pmatrix}.$$

$$\left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{array}\right) \begin{array}{l} \\ -\mathrm{I} \end{array} \longrightarrow \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -1 & 1 \end{array}\right) \begin{array}{l} -\mathrm{II} \\ \\ \end{array} \longrightarrow$$

$$\left(\begin{array}{cc|cc} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 1 \end{array}\right)$$

$$\boxed{\begin{pmatrix} 2 & -1 \\ -1 & 1 \end{pmatrix}}$$