MATH 2270
Quiz #3 - Fall 2008

Name: Answer Key

1. (3 points) If possible, compute the following matrix products. If the matrix product is undefined, write *undefined*.

   $3 \times 2 \quad 2 \times 2$

   (a) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix} = \begin{pmatrix} a & c \\ b & d \\ 0 & 0 \end{pmatrix}$

   $3 \times 1 \quad 1 \times 3$

   (b) $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{pmatrix}$

   $1 \times 3 \quad 3 \times 3 \quad 3 \times 1$

   (c) $\begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} a & d & g \\ b & e & h \\ c & f & k \end{pmatrix} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ e \\ f \end{pmatrix} = \boxed{f}$

2. (4 points) Consider the matrix

$$A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}.$$

Find *two* vectors in $\mathbb{R}^3$ that span $\ker(A)$. Clearly indicate your answer.

$$\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}\begin{matrix} \\ -I \\ -I \end{matrix} \longrightarrow \begin{pmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

two free variables, x_2, x_3

$$x_1 + x_2 + x_3 = 0$$

$$\underline{x_1 = -x_2 - x_3}$$

let $x_2 = s$

$\underline{\quad x_3 = t}$

Then $\ker(A) = \left\{ \begin{pmatrix} -s-t \\ s \\ t \end{pmatrix} \right\} = \left\{ s\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right\}$

$$\therefore \ker(A) = \operatorname{span}\left(\begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix} \right)$$

these are the two vectors

3. (4 points) True or false. Indicate whether the following statements are true or false.

(a) If A is the matrix

$$A = \overset{3\times 2}{\begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}}$$

then $\ker(A)$ is a subset of $\mathbb{R}^2$.

If $T(\vec{x}) = A\vec{x}$, then $T: \mathbb{R}^2 \to \mathbb{R}^3$. Since $\ker(A)$ is a subset of the domain of T, this is $\boxed{\text{true}}$.

(b) If A is the matrix above, then $\text{im}(A)$ is a subset of $\mathbb{R}^2$.

Similarly, $\text{im}(A)$ is a subset of the range, i.e. $\mathbb{R}^3$

So this is $\boxed{\text{false}}$

(c) If A and B are two $n \times n$ matrices, then it is always the case that

$$(A - B)(A + B) = A^2 - B^2$$

$(A-B)(A+B) = A^2 + AB - BA - B^2 \neq A^2 - B^2$

unless $AB = BA$, or A and B commute.

Since this is not always true this statement is $\boxed{\text{false}}$

(d) If A is an invertible $n \times n$ matrix, then it is always the case that

$$(I_n + A)(I_n + A^{-1}) = 2I_n + A + A^{-1}$$

$$(I_n + A)(I_n + A^{-1}) = I_n + A^{-1} + A + I_n$$

$$= 2I_n + A + A^{-1} \checkmark$$

$\boxed{\text{True}}$