MATH 2270

Quiz #6 - Fall 2008

Name: Answer Key

1. (5 points) Find the determinant of the matrix

$$A = \begin{pmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{pmatrix}$$

using any method.

<u>2 approaches</u>:

<u>#1</u>: Expand down first column:

$\det(A) =$

expand down first column again

$$1 \cdot \begin{vmatrix} 2 & 3 & 4 \\ 0 & 0 & 4 \\ 0 & 3 & 4 \end{vmatrix} = 1 \cdot 2 \cdot \begin{vmatrix} 0 & 4 \\ 3 & 4 \end{vmatrix} = 1 \cdot 2 \cdot (-12) = \boxed{-24}$$

<u>#2</u>: Use Gauss-Jordan Elimination:

$$\begin{pmatrix} 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 1 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \end{pmatrix} \sim \begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 3 & 4 \end{pmatrix} \sim \underbrace{\begin{pmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 3 & 4 \\ 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 4 \end{pmatrix}}_{} = B$$

$\det(B) = 24$, $\therefore \det(A) = (-1)^3 \cdot 24 = \boxed{-24}$

2. (4 points) Let

$$A = \begin{pmatrix} k & 2 \\ 3 & 4 \end{pmatrix}.$$

Use the determinant to calculate the values of k for which the matrix A is invertible.

$\det A = 4k - 3\cdot 2 = 4k - 6$. Then A is invertible if and only if $4k-6 \neq 0 \iff k \neq \frac{6}{4} = \boxed{\frac{3}{2}}$

i.e. A is invertible whenever $\boxed{k \neq 3/2}$

3. (2 points) True or false. Indicate whether the following statements are true or false.

(a) If all entries of a 9×9 matrix A are 9, then $\det(A) = 9^9$.

$\boxed{\text{False}}$. Clearly A is not invertible $\Rightarrow \det(A) = 0$.

(b) If A and B are two $n \times n$ matrices, then $\det(AB) = \det(BA)$.

$\boxed{\text{True}}$

$\det(AB) = \det(A)\det(B)$

$\det(BA) = \det(B)\det(A)$ ✓