Math 6010, Fall 2004: Homework

Homework 2 (Due Wednesday 15th)

(1) Consider the set

$$S = \left\{ \boldsymbol{x} \in \mathbf{R}^n : \ \frac{1}{n} \sum_{i=1}^{n} x_i = 0 \right\}.$$

    (a) Prove that S is a subspace of $\mathbf{R}^n$.

    (b) Compute the projection matrices $\mathbf{P}_S$ and $\mathbf{I}_n - \mathbf{P}_S$. Use the latter expression to find an expression for the orthogonal complement to S; i.e.,

$$S^{\perp} = \{ \boldsymbol{y} \in \mathbf{R}^n : \boldsymbol{y}'\boldsymbol{x} = 0 \text{ for all } \boldsymbol{x} \in S \}.$$

    (c) For all $\boldsymbol{y} \in \mathbf{R}^n$ compute, explicitly, the distance between $\boldsymbol{y}$ and the subspace S.

(2) Prove that $Q(x_1, x_2) = x_1 x_2$ is a quadratic form.

(3) Problem 4, page 12.