

Name. _____

Calculus III 2210-4
Sample Midterm Exam 2
Exam Date: 4 November 2005

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 75%. The answer counts 25%.

1. **(Vector geometry in \mathcal{R}^3)** Complete two of the following.
 - (a) Find the volume of a parallelepiped with edges $3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$, $-\mathbf{i} + 2\mathbf{j} + \mathbf{k}$ and $3\mathbf{i} - 2\mathbf{j} + 5\mathbf{k}$. See 14.3-18.
 - (b) Prove $|\mathbf{u} \times \mathbf{v}|^2 = |\mathbf{u}|^2|\mathbf{v}|^2 - (\mathbf{u} \cdot \mathbf{v})^2$. See 14.3-25.
 - (c) Find the parametric equation of the line through $(1, 1, 0)$ normal to the plane $2x + 4y + z = 5$.
 - (d) Find the equation of the plane through the three points $(1, 1, 0)$, $(1, 2, 0)$, $(0, 1, 1)$.

2. **(Coordinate systems)** Complete two of the following.
 - (a) Change $x^2 - y^2 = 25$ to cylindrical coordinates.
 - (b) Change $x^2 + y^2 + 2z^2 = 25$ to spherical coordinates.
 - (c) Find the center and radius of the sphere with equation $x^2 + y^2 + z^2 - 6x + 2y - 8z = 0$.

3. **(Level sets)** Complete two of the following.
 - (a) Sketch the level curves of $z = x^2/y$ for $z = -1, 1, 2$.
 - (b) Describe geometrically the level surface $16x^2 + 16y^2 - 9z^2 = k$ for all real values of k .
 - (c) Find a normal to the level curve $x^2/y = 4$ at each point (x, y) .

4. **(Derivatives)** Complete two of the following.
 - (a) Find f_x, f_{xy} for $f = (x^3 + y^2)^5$.
 - (b) Let $f(x, y) = xy^2/(x + 2 + y^4)$. Is f continuous at $(0, 0)$? Discuss continuity of the partial derivative f_{xy} .
 - (c) Define f_x using Newton quotients and limits.
 - (d) Find the gradient of $f(x, y, z) = (x + 2y + 3z)^2 e^{3x+4y+5z}$ at $x = y = z = 0$.

5. **(Chain rule)** Complete two of the following.
 - (a) Suppose $x \sin y + y \cos x = 10$. Find a formula for dy/dx .
 - (b) Let $w = e^{x^2+y^2}$, $x = t^3$, $y = t^2$, $z = t$. Find dw/dt .
 - (c) Let $w = u/v$, $u = x^2 - 3y$, $v = xyz$. Find the partials of w in variables x, y, z .