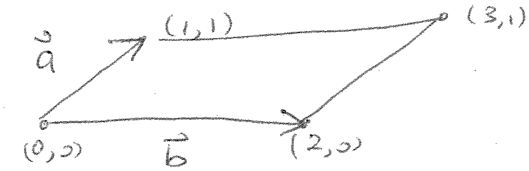
Exam 2
2210-4

Name. KEY

1. **(Vector geometry in $\mathcal{R}^3$)** Complete both of the following.
(a) [75%] Find the volume of the cylinder (a parallelepiped) formed from the planar parallelogram with vertices $(0,0)$, $(1,1)$, $(2,0)$, $(3,1)$ by dragging the parallelogram distance 2 in the direction of the vector $\mathbf{3i} - \mathbf{2j} + \mathbf{5k}$.
(b) [25%] Find the parametric equation of the line through $(1,0,-1)$ and parallel to the line of intersection of the two planes $x+y-z=1$, $2x+y+z=2$.



ⓐ $\pm(\text{volume}) = \vec{a}\cdot\vec{b}\times\vec{c}$

$$= \frac{2}{\sqrt{38}}\begin{vmatrix} 1 & 1 & 0 \\ 2 & 0 & 0 \\ 3 & -2 & 5 \end{vmatrix}$$

$$= -\frac{4}{\sqrt{28}}\begin{vmatrix} 1 & 0 \\ -2 & 5 \end{vmatrix}$$

$$= \frac{-20}{\sqrt{38}}$$

$\vec{a} = \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}$, $\vec{b} = \begin{pmatrix} 2 \\ 0 \\ 0 \end{pmatrix}$,

$\vec{c} = 2\dfrac{\vec{d}}{|\vec{d}|}$, $\vec{d} = \begin{pmatrix} 3 \\ -2 \\ 5 \end{pmatrix}$

$|\vec{d}| = \sqrt{9+4+25}$
$= \sqrt{38}$

$\boxed{\text{vol} = \dfrac{20}{\sqrt{38}}}$

ⓑ $\vec{n} = \vec{n}_1 \times \vec{n}_2$ $\qquad \vec{n}_1 = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$, $\vec{n}_2 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$

$\vec{n} = \vec{n}_1 \times \vec{n}_2$

$$= \begin{vmatrix} i & j & k \\ 1 & 1 & -1 \\ 2 & 1 & 1 \end{vmatrix}$$

$$= \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 2t+1 \\ -3t+0 \\ -t+(-1) \end{pmatrix}$$

$\boxed{\vec{x} = t\begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} + \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}}$ or $\boxed{\begin{aligned} x &= 1+2t \\ y &= -3t \\ z &= -1-t \end{aligned}}$

vector form $\qquad$ scalar form

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5-problem KEY written in 16 min

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2. (Coordinate systems) Complete both of the following.
(a) [50%] Change the cylindrical coordinate equation $r^2 \cos\theta - \sin\theta\cos\theta + r^4 = 1 + z$ to standard xyz-coordinates.
(b) [50%] Find the three semi-axis lengths of the ellipsoid with equation $x^2 + 4y^2 + z^2 - 6x + 8y - 8z = 0$.

(a) $\begin{cases} x = r\cos\theta \\ y = r\sin\theta \\ z = z \end{cases}$

$r^2\cos\theta - \sin\theta\cos\theta + r^4 = 1+z$

$rx - \frac{y}{r}\frac{x}{r} + r^4 = 1+z$

$$\boxed{x\sqrt{x^2+y^2} - \frac{xy}{x^2+y^2} + (x^2+y^2)^2 = 1+z}$$

(b) $x^2 - 6x + 4(y^2+2y) + z^2 - 8z = 0$

$(x-3)^2 + 4(y+1)^2 + (z-4)^2 - 9 - 4 - 16 = 0$

$(x-3)^2 + 4(y+1)^2 + (z-4)^2 = 29$

$\frac{(x-3)^2}{a^2} + \frac{(y+1)^2}{b^2} + \frac{(z-4)^2}{c^2} = 1$

$a^2 = 29$
$b^2 = 29/4$
$c^2 = 29$

$$\boxed{a = \sqrt{29},\quad b = \sqrt{29}/2,\quad c = \sqrt{29}}$$

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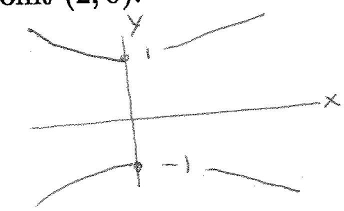
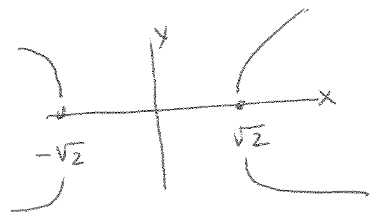
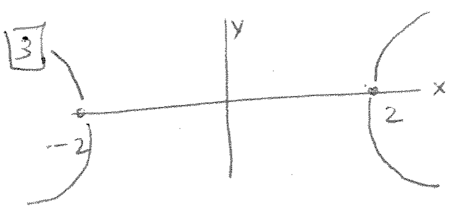
3. (Level sets) Complete both of the following.
(a) [50%] Sketch the level curves of $z = x^2 - y^2$ for $z = -1, 2, 4$.
(b) [50%] Find a normal to the level curve $x^2 - 2y^2 = 4$ at the point $(2, 0)$.

(a)
$\boxed{1}$ $x^2 - y^2 = -1$ or $y^2 - x^2 = 1$
$\boxed{2}$ $x^2 - y^2 = 2$
$\boxed{3}$ $x^2 - y^2 = 4$

$\boxed{1}$


$\boxed{2}$


$\boxed{3}$


(b) $f(x,y) = x^2 - 2y^2 - 4 = 0$

$$\text{grad } f = \begin{pmatrix} 2x \\ -4y \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$\text{normal} = \frac{\text{grad}(f)}{|\text{grad}(f)|}$$

$$\boxed{= \begin{pmatrix} 1 \\ 0 \end{pmatrix} = \vec{i}}$$

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4. (Derivatives) Complete both of the following.
(a) [50%] Find $f_x(1,1)$ for $f(x,y) = (2x^3+3y^2)^4(3x+2y)^2$.
(b) [50%] Find the directional derivative of $f(x,y) = (x+2y+1)^2 e^{3x+4y}$ at $x = y = 0$ in the direction of the vector $\mathbf{i} - 2\mathbf{j}$.

(a)
$$f_x = 4(2x^3+3y^2)^3(3x+2y)^2(6x^2) + (2x^3+3y^2)^4(2)(3x+2y)(3)$$
$$= 4(5)^3(5)^2(6) + (5)^4(2)(5)(3)$$
$$= 5^5(24+6)$$
$$= 30 \cdot 5^5$$
$$= \boxed{6 \cdot 5^6} \text{ or } \boxed{93750}$$

(b)
$$\vec{u} = \begin{pmatrix} 1 \\ -2 \end{pmatrix} \cdot \frac{1}{\sqrt{5}} \quad \text{a unit vector}$$

$$\operatorname{grad}(f) = \begin{pmatrix} 2(x+2y+1)e^{3x+4y} + (x+2y+1)^2 e^{3x+4y} \cdot 3 \\ 2(x+2y+1)e^{3x+4y}(2) + (x+2y+1)^2 e^{3x+4y}(4) \end{pmatrix}$$
$$= \begin{pmatrix} 2+3 \\ 4+4 \end{pmatrix}$$
$$= \begin{pmatrix} 5 \\ 8 \end{pmatrix}$$

$$DD = \begin{pmatrix} 5 \\ 8 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ -2 \end{pmatrix} \frac{1}{\sqrt{5}}$$
$$= (5-16)/\sqrt{5}$$
$$= \boxed{\frac{-11}{\sqrt{5}}}$$

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5. (Chain rule) Complete both of the following.
(a) [50%] Consider $x \sin y + ye^x = 10$. Find a formula for dy/dx.
(c) [50%] Let $w = u/v$, $u = x^2 + xy$, $v = x^2y^3 + x^3$. Find the partials w_x and w_y.

(a)

$F(x,y) = 10$

$F_x dx + F_y dy = 0$

$\frac{dy}{dx} = -\frac{F_x}{F_y}$

$F_x = \sin y + ye^x$

$F_y = x\cos y + e^x$

$$\boxed{\frac{dy}{dx} = -\frac{\sin y + ye^x}{x\cos y + e^x}}$$

(b)

$$w_x = \begin{pmatrix} w_u \\ w_v \end{pmatrix} \cdot \begin{pmatrix} u_x \\ v_x \end{pmatrix}$$

$$= \begin{pmatrix} 1/v \\ -u/v^2 \end{pmatrix} \cdot \begin{pmatrix} 2x+y \\ 2xy^3 + 3x^2 \end{pmatrix}$$

$$= \frac{2x+y}{v} + \frac{(2xy^3 + 3x^2)(-u)}{v^2}$$

$$= \boxed{\frac{2x+y}{x^2y^3 + x^2} + \frac{(2xy^3 + 3x^2)(-x^2 - xy)}{(x^2y^3 + x^3)^2}}$$

$$w_y = \begin{pmatrix} 1/v \\ -u/v^2 \end{pmatrix} \cdot \begin{pmatrix} x \\ 3x^2y^2 \end{pmatrix}$$

$$= \boxed{\frac{x}{x^2y^3 + x^3} + \frac{3x^2y^2(-x^2 - xy)}{(x^2y^3 + x^3)^2}}$$

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