Math 5110 - Fall 2012
Homework Problem Set 6
Due Nov. 20, 2007

1. Suppose that a flu virus (an SIR disease) sweeps through a city of a million people affecting 40% of the population.
   (a) Estimate the critical population size γ (i.e., the size necessary to have an epidemic). (Include plots of the solution for this value of γ.
   (b) At the height of the epidemic, what percentage of the population is infected?
   (c) Predict the outcome of this disease for a community of 500,000.

2. Krill are small shrimp-like animal that are the main food source for the baleen whale in the southern ocean. A possible model for the these interacting populations is

$$\frac{dS}{dt} = S(1-S) - \frac{SW}{K+S} \tag{1}$$
$$\frac{dW}{dt} = \frac{bSW}{K+S} - rW. \tag{2}$$

   (a) Describe the meaning of each of the terms and why they are reasonable.
   (b) Find the parameter range for which there is a stable positive steady state solution and the parameter range for which there is an unstable positive steady state solution.
   (c) Find the numerical solution and a phase portrait of the solution (include plots of the nullclines) for values of the parameters in each of these two parameter ranges.
   (d) Describe the effect of harvesting krill.
   (e) Describe the effect of harvesting whales.

3. The purpose of this problem is to study the behavior of the chemostat equations for a renewable resource with two organisms competing for the same resource,

$$\frac{dS}{dt} = S(1-S) - \frac{m_1 SU}{K_1+S} - \frac{m_2 SV}{K_2+S} \tag{3}$$
$$\frac{dU}{dt} = \frac{m_1 SU}{K_1+S} - U \tag{4}$$
$$\frac{dV}{dt} = \frac{m_2 SV}{K_2+S} - V \tag{5}$$

   Specifically, the goal is find when coexistence of U and V is possible.
   (a) Show that coexistence is not possible if both U and V are steady.
   (b) Set $K_1 = 0.5$ and $K_2 = 0.25$. Describe in words what this implies about the difference between the two species U and V.
   (c) For what range of m_1 does U have a stable oscillation when in isolation, and for what range of m_2 does V have a stable oscillation when in isolation?
   (d) **For this and the remainder of this exercise, use numerical simulations to establish the answer.** Suppose $m_1 = m_2$. Which species survives, and why?

(e) Coexistence is possible only if $m_1 > m_2$. Why?

(f) Suppose $m_1 = 3.5$. Find a range of values of m_2 for which coexistence is possible (if there is one). What is the behavior of U in isolation?

(g) Suppose $m_1 = 2.8$. Find a range of values of m_2 for which coexistence is possible (if there is one). What is the behavior of U in isolation?

(h) Summarize your findings by describing the characteristics of the growth functions that are necessary in order to permit coexistence. (Make a plot of the growth functions to help illustrate your point.)