Math1010
Equation Solving Strategies

(1) *Linear Equations* —Simplify both sides of the equation, isolate the variable.

Example:

$$3(x-2)+1=5x-7$$
$$3x-6+1=5x-7$$
$$3x-5=5x-7$$
$$-5=2x-7$$
$$2=2x$$
$$1=x$$

(2) *Quadratic Equations*—

   (a) If it has an x term in it, then move everything on one side of the equation with the other side zero. Try to factor it.

      1. If it factors, set each factor equal to zero and solve.

      2. If it doesn't factor, use the quadratic formula, i.e. For $ax^2+bx+c=0$, $x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$, or you can complete the square.

   (b) If it doesn't have an x term in it, then isolate the x^2 and take the square root of both sides. (Remember to put the plus or minus sign on the answer.)

Examples:

$$3x^2+13x=10$$
$$3x^2+13x-10=0$$
$$(3x-2)(x+5)=0$$

(a) 1.
$$3x-2=0 \text{ or } x+5=0$$
$$3x=2 \text{ or } x=-5$$
$$x=\frac{2}{3} \text{ or } x=-5$$

(a) 2.
$$2x^2+x-5=0$$
$$a = 2, b = 1, c = -5$$
$$x=\frac{-1\pm\sqrt{1-4(2)(-5)}}{2(2)}$$
$$x=\frac{-1\pm\sqrt{1+40}}{4}$$
$$x=\frac{-1\pm\sqrt{41}}{4}$$

(b)
$$2x^2-50=0$$
$$2x^2=50$$
$$x^2=25$$
$$x=\pm5$$

(Note: For quadratic equations, the quadratic formula method will ALWAYS work. So, if you'd like, you can always use that method.)

(3) *Higher-Order Polynomial Equations*—Get everything on one side of the equation, with the other side zero. Factor and then set each factor equal to zero and solve. (This is basically a more intense version of the 2(a)1. type.)

$$10x^4 = 5x^3 + 15x^2$$
$$10x^4 - 5x^3 - 15x^2 = 0$$
$$5x^2(2x^2 - x - 3) = 0$$

Example:
$$5x^2(2x - 3)(x + 1) = 0$$
$$5x^2 = 0 \quad \text{or} \quad 2x - 3 = 0 \quad \text{or} \quad x + 1 = 0$$
$$x = 0 \quad \text{or} \quad x = \frac{3}{2} \quad \text{or} \quad x = -1$$

(4) *Rational Equations*—Find the least common denominator (LCD). Multiply both sides of the equation by the LCD which will get rid of the fractions. Then, it turns into either a linear or quadratic equation and you solve those using the previously stated methods.

$$\frac{1}{x+1} - \frac{2}{x+5} = \frac{x+2}{x+1}$$

LCD = (x+1)(x+5)

Example:
$$\frac{(x+1)(x+5)}{x+1} - \frac{2(x+1)(x+5)}{x+5} = \frac{(x+2)(x+1)(x+5)}{x+1}$$
$$x+5 - 2(x+1) = (x+2)(x+5)$$
$$x+5 - 2x - 2 = x^2 + 7x + 10$$
$$-x + 3 = x^2 + 7x + 10$$
$$0 = x^2 + 8x + 7$$
$$0 = (x+7)(x+1)$$
$$0 = x+7 \quad \text{or} \quad 0 = x+1$$
$$x = -7 \quad \text{or} \quad x = -1$$

(**Note**: You MUST check your answers for rational equations to make sure they don't give you a zero in the denominator of the original equation. If a solution makes the denominator go to zero, then you throw out that solution.)

(5) *Radical Equations*—Isolate the radical expression. If there are two radical expressions, isolate one of them (either one). Then raise both sides of the equation to the power that will "undo" the radical. So, if it's a square root, then you would square both sids. If it's a cube root, then cube both sides. If there is still a radical expression left in the equation, repeat   this process, until there are no radical expressions left. Then, the equation will either be a linear or quadratic equation and you can solve appropriately.

Example:

$$\sqrt{x-6}+3=\sqrt{x+9}$$
$$\left(\sqrt{x-6}+3\right)^2=\left(\sqrt{x+9}\right)^2$$
$$\left(\sqrt{x-6}+3\right)\left(\sqrt{x-6}+3\right)=x+9$$
$$(x-6)+3\sqrt{x-6}+3\sqrt{x-6}+9=x+9$$
$$x-6+9+6\sqrt{x-6}=x+9$$
$$x+3+6\sqrt{x-6}=x+9$$
$$6\sqrt{x-6}=6$$
$$\sqrt{x-6}=1$$
$$\left(\sqrt{x-6}\right)^2=1^2$$
$$x-6=1$$
$$x=7$$

(**Note**: If your original equation has an even root in it, then you MUST check your solution by plugging it into the original equation to see if it really works. If it doesn't work, then throw it out.)

(6) *Exponential Equations*—Isolate the exponential expression on one side of the equation. Then, rewrite the equation as its corresponding logarithmic equation using the definition of Logarithm. Then, the equation will either be a linear or quadratic equation and you can solve appropriately.

Example:

$$\frac{50}{2-3e^{2x-5}}=400$$

$$(2-3e^{2x-5})\left(\frac{50}{2-3e^{2x-5}}\right)=400(2-3e^{2x-5})$$

$$50=400(2-3e^{2x-5})$$

$$\frac{50}{400}=2-3e^{2x-5}$$

$$\frac{1}{8}-2=-3e^{2x-5}$$

$$\frac{-7}{8}=-3e^{2x-5}$$

$$\frac{-1}{3}\cdot\frac{-7}{8}=e^{2x-5}$$

$$\frac{7}{24}=e^{2x-5}$$

$$\ln\left(\frac{7}{24}\right)=2x-5$$

$$\ln\left(\frac{7}{24}\right)+5=2x$$

$$\frac{1}{2}\left(\ln\left(\frac{7}{24}\right)+5\right)=x$$

(7) Logarithmic Equations—Isolate the logarithmic expression on one side of the equation. If there are several logarithmic terms, you'll need to use logarithmic properties to condense them into one logarithmic expression first. Then, rewrite the equation as its corresponding exponential form using the definition of Logarithm. The equation will most likely either be a linear or quadratic equation to finish solving.

Example:

$$2\log_4 x - 3 = \log_4(x-16)$$
$$2\log_4 x - \log_4(x-16) = 3$$
$$\log_4 x^2 - \log_4(x-16) = 3$$
$$\log_4\left(\frac{x^2}{(x-16)}\right) = 3$$
$$4^3 = \frac{x^2}{(x-16)}$$
$$4^3(x-16) = x^2$$
$$64x - 64(16) = x^2$$
$$0 = x^2 - 64x + 1024$$
$$0 = (x-32)(x-32)$$
$$x - 32 = 0$$
$$x = 32$$

(**Note**: For logarithmic equations you MUST check your solution by plugging it into the original equation to see if it is in the domain of the logarithm. We can only take the log of positive numbers, so if our proposed solution makes us try to take the log of a negative number or zero, then we must throw out that solution.)