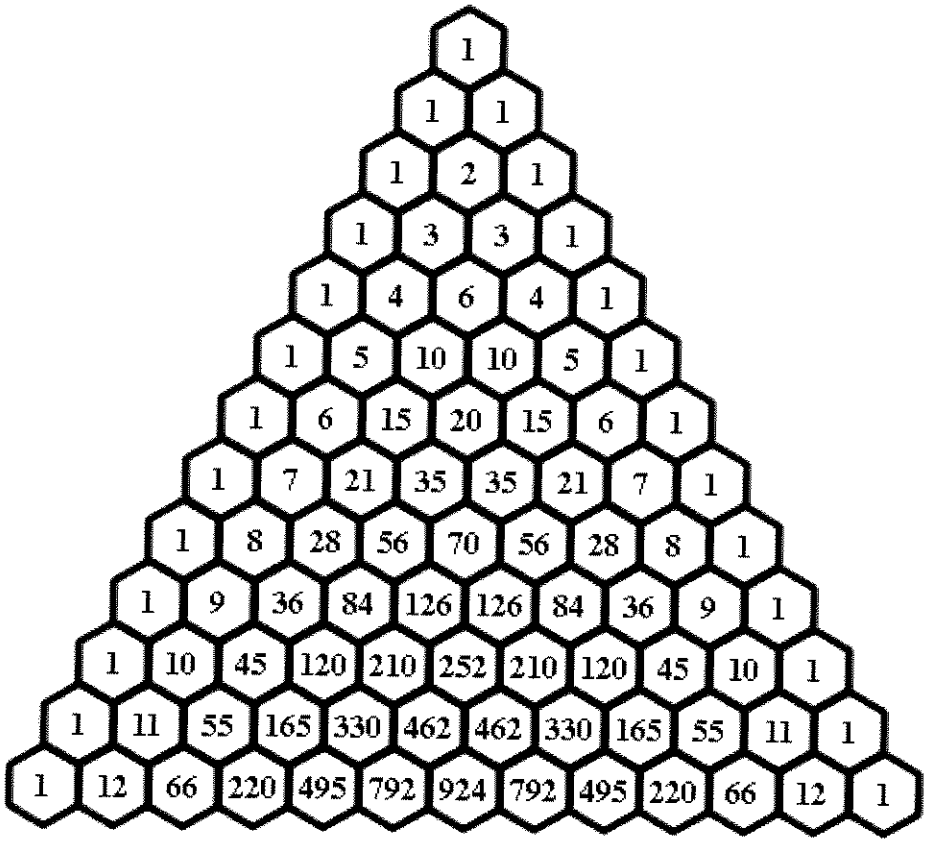
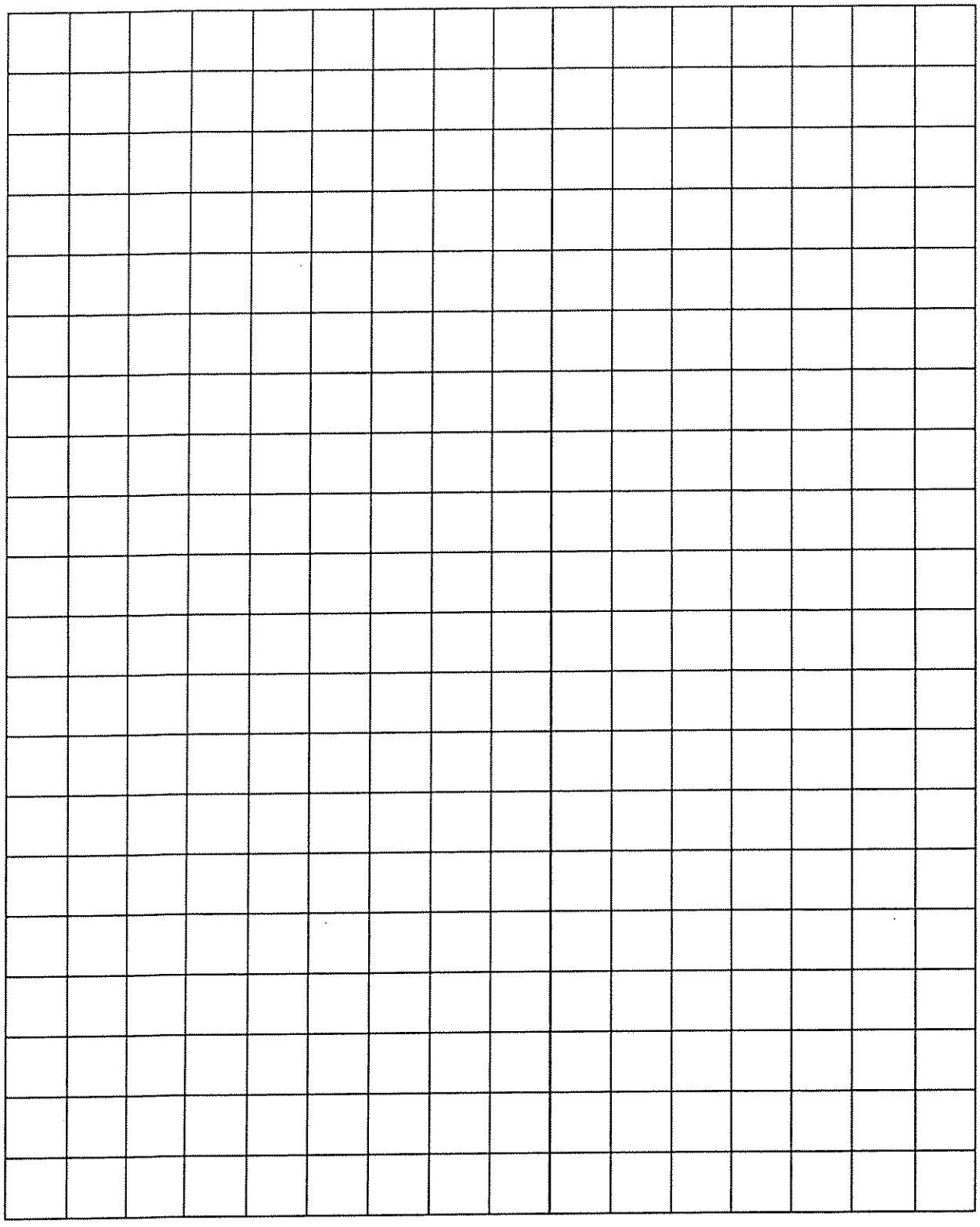
# Discovering Patterns

**Name** _____





Teachers need to become familiar with the pleasures of figuring things out, as well as with the concomitant experiences of confusion and frustration, tolerating their discomfort long enough for things to fall into place. If teachers never learn what this experience feels like, they won't have the gumption to allow their students to go through it either.

CBMS vol. 11

The Dean of the college of Science.... Maintains that "nationally college students are advised to study at least two hourse per week per unit of coursework. AT CalPoly we endorse using this as a minimum estimate forr planning your time and suggest you allocate more for science and mathematics clases. Thus, as Math 424 is a 4-unit course, you should plan to read and write a minimum of 8 to 12 hours per week for this class.

---

4010 Number activity

Organize these numbers into sets and justify why you organized them in the way you did.

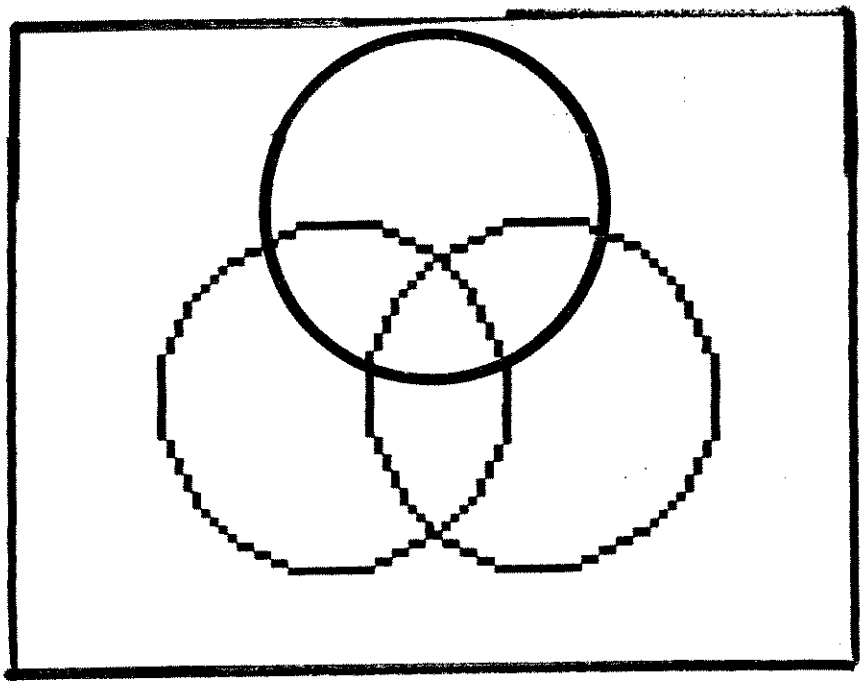
| | | | | | |
|---|---|---|---|---|---|
| 0 | 97 | .78 | $0.\overline{9}$ | .142857142857.... | |
| 3/4 | $\pi$ | 38% | $.\overline{43}$ | -1 | |
| $.\overline{2}$ | .121121112..... | $\frac{22}{7}$ | $\sqrt{16}$ | $-\frac{2}{3}$ | 1 |
| -.32 | $\frac{8}{2}$ | 3.14 | $\sqrt{2}$ | $\sqrt{-5}$ | |

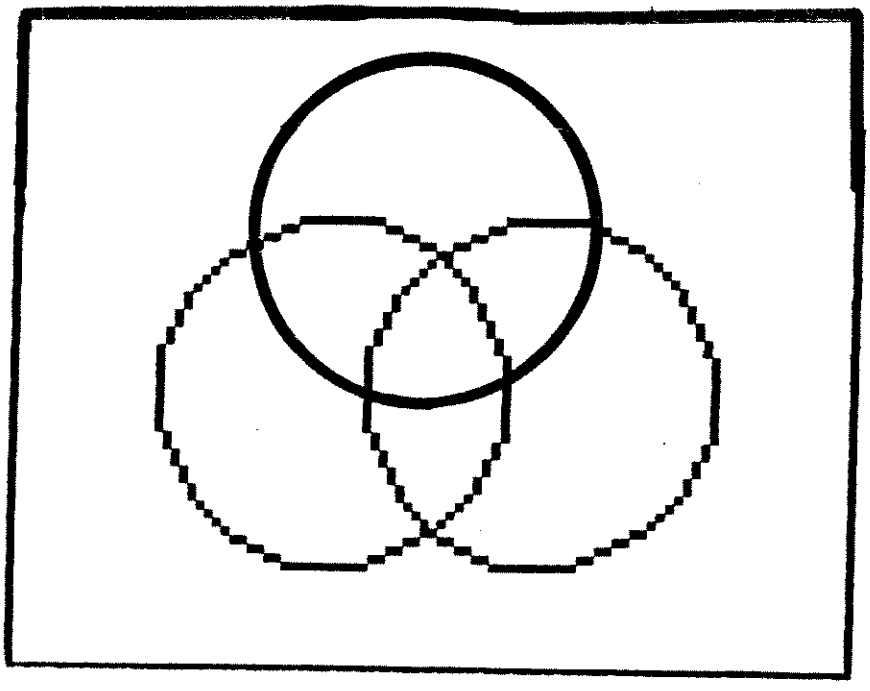
**Why do I have to take this class?**

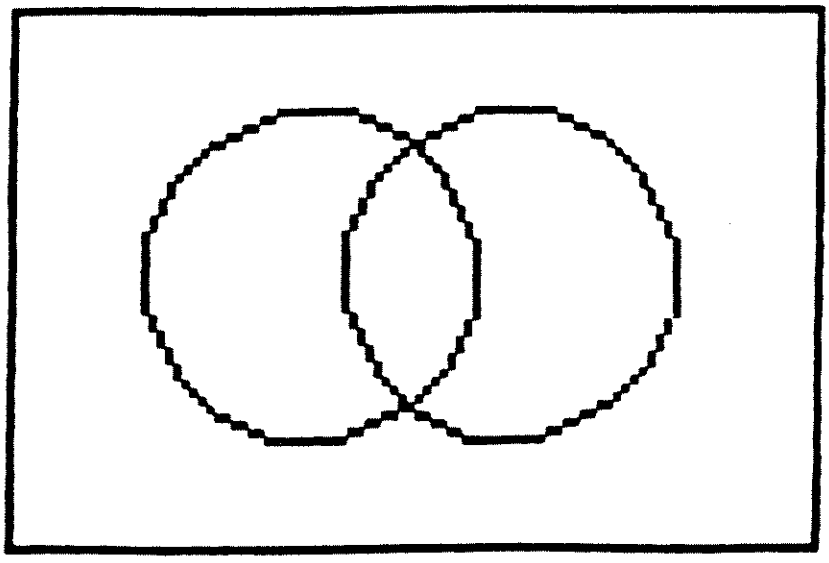
- In order to teach we need to have a deeper understanding of that which we teach.
- We need to see where what we teach fits into the bigger picture.
- Teachers must be able to do more than demonstrate remembered procedures. They must be able to select problems that anticipate the issues their students will next confront, then assess whether what the children make of those problems advances the mathematical agenda. Such skills require much deeper understanding of number and operation than many teachers now hold. (CBMS)
- US behind in math and science. WHY?
- We must have a fundamental understanding of the entire curriculum so we are ready to exploit opportunities to review concepts students have previously studied or to lay the groundwork for a concept to be studied later. (L. Ma)

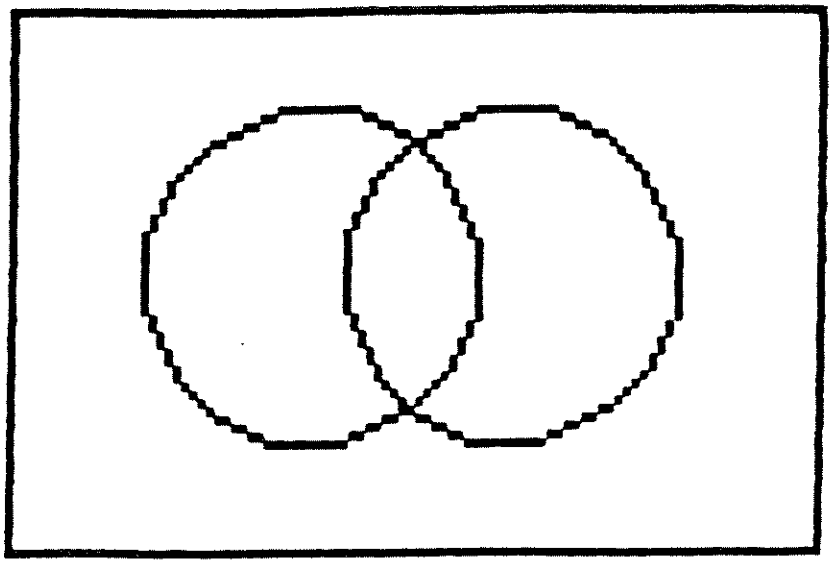
**Examples of where and how teachers use mathematics for teaching from D. Ball**

- Clarifying mathematical goals and approaches
- Communicating with teachers of other grade levels, principals, specialists
- Deciding what to take up and what to leave in a discussion
- Designing lessons
- Evaluating explanations, arguments, proofs
- Examining students' work
- Explaining the curriculum to parents
- Interpreting curriculum materials
- Interpreting, using, and managing state curriculum and assessment policies
- Listening to students
- Making homework assignments
- Posing questions
- Setting up mathematical tasks
- Writing quizzes
- Writing, representing, and recording mathematics

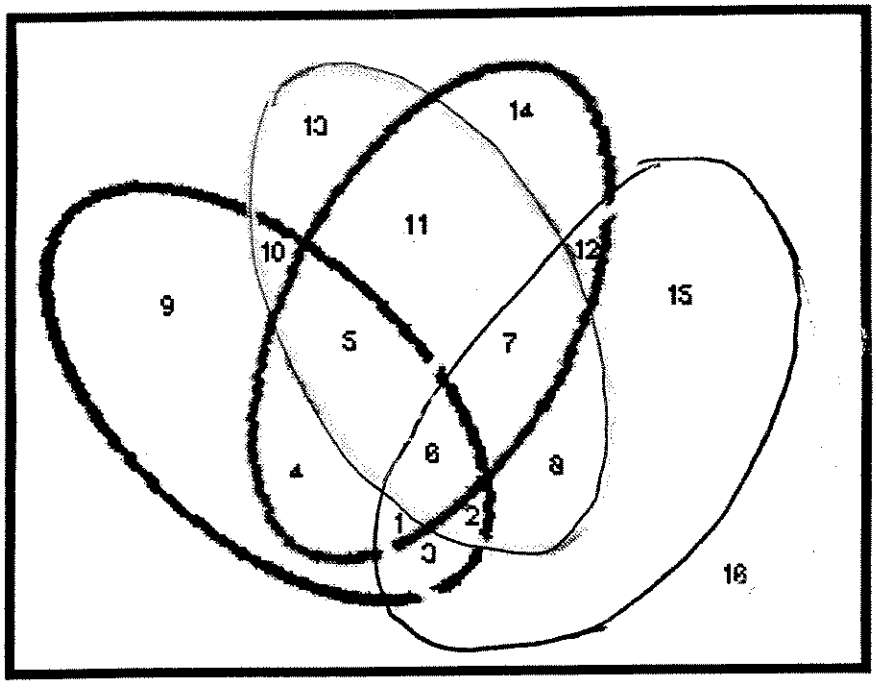








venn4set.gif (GIF Image, 263x201 pixels)



8

# More Venn Diagram Exercises

Use a Venn Diagram and partitions to summarize and analyze the data in each problem then use it to answer the questions. Submit your answers in tabular form showing the results in set operation format for the sets and the partitions that comprise each answer.

1) Toward the middle of the season' peaches for canning tend to come in three types: Early, late and extra late, depending on the expected date of ripening. During one week, the following data were recorded at a small peach receiving station:

- 16 trucks were dispatched carrying early peaches
- 36 trucks had late peaches
- 33 trucks had extra late peaches
- 13 trucks had early and late peaches
- 15 trucks had late and extra late peaches
- 1 truck had early and extra late peaches
- no trucks had all three types

Determine the number of trucks:

a) carrying only late peaches (8)
b) carrying only one variety of peaches (27)
c) carrying exactly two varieties of peaches (29)
d) the total number of trucks (56)

2) A group of 195 people were polled to see if they watched certain TV programs, which we will refer to as programs A, B and C. The results were:

- 39 watched A
- 90 watched B
- 51 watched C
- 10 watched all three
- 16 watched both B and C
- 30 watched C only
- 14 watched both A and B

Determine how many:

a) did not watch any of the three (50)
b) watched at least two programs (25)
c) watched either B or C (125)
d) both A and C (15)

8

9

3) A survey of 100 Atlanta residents was taken to determine how well they liked the Braves, the Falcons and the Hawks. It was found that:

- 63 liked the Braves
- 62 did not like the Falcons
- 18 did not like the Falcons or the Hawks
- 30 liked the Falcons and the Braves
- 28 did not like the Hawks
- 20 like all three
- 15 did not like any of the three

Determine how many like:

a) only the Hawks
b) both the Hawks and the Falcons
c) at least two of the three

9

10

Venn Diagram Exercises page 1 of 2

Use a Venn Diagram and ~~partitions~~ to summarize and analyze the data in each problem so you can answer the questions.

1) Five hundred men were asked which of three grocery stores (Kroger, Publix, Costco) at which they shopped. The results were are follows:

- 230 shop at Kroger
- 115 shop at Publix
- 170 shop at Costco
- 5 shop at all three
- 20 shop at Kroger and Costco
- 15 shop at Costco and Publix
- 35 shop at Kroger and Publix

Determine how many of these men shop at:

a) Kroger Only
b) Costco and Publix only
c) Kroger and Costco
d) either Costco or Publix
e) exactly one of the three stores
f) at least two of the three stores
g) none of the three stores

2) A survey of 180 people showed that:

- 60 like hamburgers
- 95 like chicken
- 120 like pizza
- 55 like pizza but not chicken
- 45 like hamburgers and pizza
- 10 like hamburgers only
- 30 like all three

Determine how many:

a) do not like any of the three (20)
b) like chicken only (25)
c) like at least one of the three (160)
d) like hamburger and chicken (35)
e) like hamburger and chicken but not pizza (5)
f) like exactly one of the three (75)
g) like either pizza or chicken (150)
h) like exactly two of the three (55)

10

Venn Diagram Exercises page 2 of 2

3) In a certain math course a survey was taken. It showed that:

450 passed the course

10 of those who failed still liked the course

25 of those who failed signed up for another math course

55 of those who liked the course signed up for another math course

60 of those who passed the course signed up for another math course

350 of those who passed the course liked it

300 of those who passed the course liked it but didn't sign up for another course

130 didn't like the course

Determine how many:

a) were surveyed (490)
b) liked the course (360)
c) didn't pass the course (40)
d) of those who failed the course, disliked it and didn't sign up for another math course (10)
e) of those who did not like the course passed it (100)
f) passed the course, liked it, and signed up for another math course (50)

four-fact family

5+3 = 8
3+5 = 8
8-5 = 3
8-3 = 5

Look at all the problems this family produces.

12

**TABLE 4.1.** A Taxonomy of Addition and Subtraction Word Problems

| CHANGE-ADD-TO with | . . . UNKNOWN OUTCOME | . . . UNKNOWN CHANGE | . . . UNKNOWN START |
|---|---|---|---|
| | Alexi had 5 candies. Barb gave him 3 more. How many candies does he have altogether now? | Alexi had 5 candies. Barb gave him some more. Now he has 8 altogether. How many candies did Barb give him? | Alexi had some candies. Barb gave him 3 more. Now he has 8 altogether. How many candies did he start with? |
| CHANGE-TAKE-AWAY with | . . . UNKNOWN OUTCOME | . . . UNKNOWN CHANGE | . . . UNKNOWN START |
| | Alexi had 8 candies. He gave 5 to Barb. How many candies does he have left? | Alexi had 8 candies. He gave some to Barb. Now he has 3 left. How many candies did he give to Barb? | Alexi had some candies. He gave 5 to Barb. Now he has 3 left. How many candies did he start with? |
| PART-PART-WHOLE with | . . . UNKNOWN WHOLE | . . . UNKNOWN SECOND PART | . . . UNKNOWN FIRST PART |
| | Alexi had 5 fireballs and 3 lollipops. How much candy did he have altogether? | Alexi had 5 fireballs and some lollipops. He had 8 candies altogether. How many were lollipops? | Alexi had some fireballs and 3 lollipops. He had 8 candies altogether. How many were lollipops? |
| EQUALIZE with | . . . UNKNOWN DIFFERENCE | . . . UNKNOWN SECOND PART | . . . UNKNOWN FIRST PART |
| | Alexi had 8 candies. Barb had 5. How many more does Barb have to buy to have as many as Alexi? | Alexi had 8 candies. Barb had to get 3 more candies to have the same number as Alexi. How many candies did Barb start with? | Alexi had some candies. Barb, who had 5 candies, had to get 3 more to have the same number as Alexi. How many candies did Alexi have? |
| COMPARE with | . . . UNKNOWN DIFFERENCE | . . . UNKNOWN SECOND PART | . . . UNKNOWN FIRST PART |
| | Alexi had 8 candies. Barb had 5. How many more candies did Alexi have than Barb? | Alexi had 8 candies. He had 3 more than Barb. How many candies did Barb have? | Alexi had some candies. He had 3 more than Barb who had 5. How many candies did Alexi have? |

*Note.* The examples shown above for EQUALIZE and COMPARE problems are the "more" versions. "Less" versions could also be written for each. For example, the less version of the EQUALIZE with UNKNOWN DIFFERENCE would read: Alexi had 8 candies. Barb had 5. How many does Alexi have to give up to have as many as Barb?

# Catch of the Day

Name _____

You are going fishing and must catch a specified number of fish each day. All "catches" must include the following fish: snapper, grouper, and tuna. The captain of the fishing boat has determined how many of each of the three kinds of fish are needed and has provided clues for the crew to use to figure out how many of each type of fish must be caught. You may use colored chips or cubes to represent the fish caught.

## Monday's Catch of the Day

3 snapper
2 more tuna than snapper
15 fish in all

S = _____ T = _____ G = _____

## Tuesday's Catch of the Day

6 snapper
1/2 as many tuna as grouper
12 fish in all

S = _____ T = _____ G = _____

## Wednesday's Catch of the Day

3 snapper
4 times as many tuna as snapper
23 fish in all

S = _____ T = _____ G = _____

## Thursday's Catch of the Day

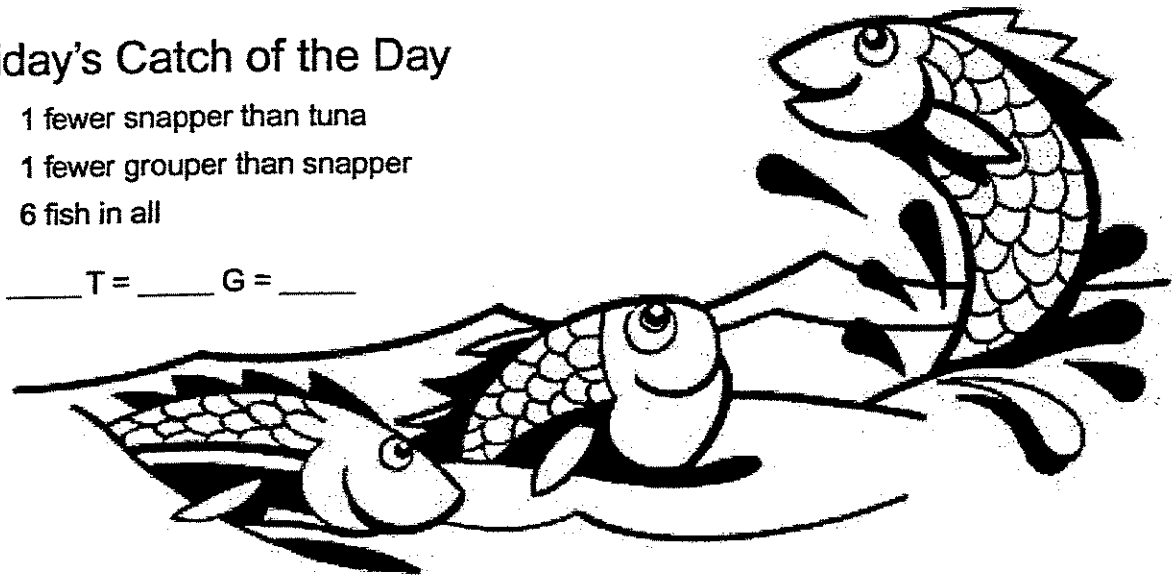
1/4 of the catch is snapper.
1/3 of the catch is grouper.
12 fish in all

S = _____ T = _____ G = _____

## Friday's Catch of the Day

1 fewer snapper than tuna
1 fewer grouper than snapper
6 fish in all

S = _____ T = _____ G = _____



## EXERCISES

# Proper-T-Practice

Each of the following computations could be simplified by applying one of the properties listed at the right below. For each computation, identify the property or properties used and place the code letter on the line in front of the computation.

___ 1. $(96 + 56) + 44 = 196$

___ 2. $(56 \times 29) + (56 \times 71) = 5600$

___ 3.
___ 4. } $132 + (51 + 68) = 251$

___ 5. $4 \times (250 \times 29) = 29000$

___ 6. $21 + (39 + 0) = 60$

___ 7. $(121 \times 49) - (21 \times 49) = 4900$

___ 8.
___ 9. } $8 \times (57 \times 125) = 57000$

___10. $(56 \times 1) \times 4 = 224$

___11. $(46 \times 27) + (54 \times 27) = 2700$

Properties of Whole Numbers

(C) commutativity for addition

(M) associativity for addition

(E) identity for addition

(H) commutativity for multiplication

(A) associativity for multiplication

(I) identity for multiplication

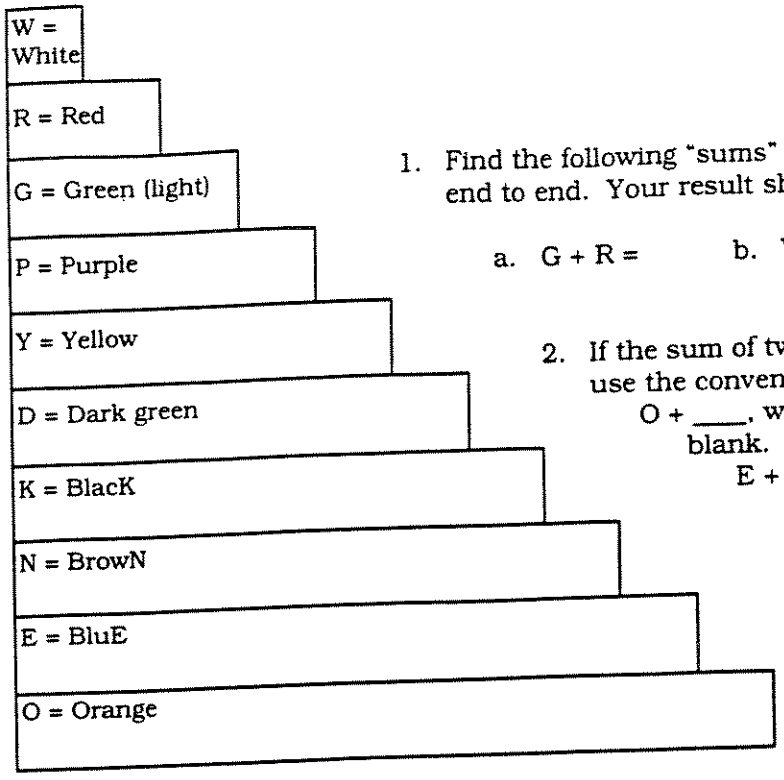
(T) distributivity for multiplication over addition

(S) distributivity for multiplication over subtraction

Now unscramble these 11 code letters to identify a subject that is an art and a tool, as well as a science.

___ ___ ___ ___ ___ ___ ___ ___ ___ ___ ___

**OBJECTIVE:** **Compute sums using a measurement model**




1. Find the following "sums" by placing your centimeter strips end to end. Your result should be a color.

   a. G + R = b. W + Y = c. P + Y =

2. If the sum of two strips exceeds O(range), we will use the convention of expressing the answer as O + ____, where the 'excess' color goes in the blank. For example,
   E + G = O + R. (Verify this!)

   Find these sums:

   a. K + E =

   b. N + D =

   c. K + K =

---

1. Find the following "sums" with your centimeter strips. Again, your result should be a color.

   a. K + N = b. N + K =
   c. Y + R = d. R + Y =
   e. G + D = f. D + G =

2. What relationship do you find between (a) and (b), (c) and (d), and (e) and (f)? Explain.

3. Fill in the portion of this table above the dotted line.

4. Use your discovery in part 2 to complete the rest of the table.

| + | W | R | G | P | Y |
|---|---|---|---|---|---|
| W | R | G | | | D |
| R | | | | | |
| G | | | | | |
| P | | | | | |
| Y | | | | | |

---

1. Compute W + R. To this result add Y. We write this as (W + R) + Y where the parentheses tell you which part to add first. In summary, (W + R ) + Y = _____ .

2. Now consider W + (R + Y). Which sum do you find first? Record your result. How does this result compare with part 1?

3. Compute these sums.

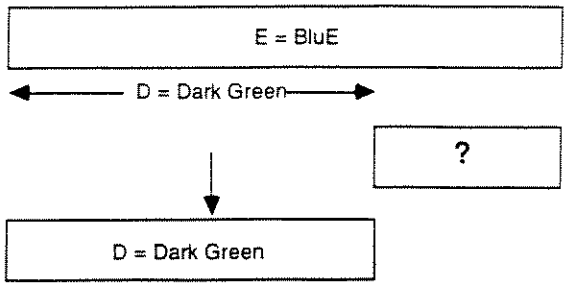
   a. (P + R) + G = b. P + (R + G) =
   c. (R + G) + Y = d. R + (G + Y) =

   What do you notice?


**OBJECTIVE:** **Represent subtraction using a measurement model**

**You Will Need:** Your centimeter strips (Activity 3.1)

1. To build a bird house, Maxime needs a board 25 cm long which he will cut from a larger board. This **"take-away" approach** can be modeled with your centimeter strips. For example, imagine that a blue strip is the larger board and a dark green centimeter strip represents the length he will cut off. If the blue strip were shortened by taking away the length of the dark green as pictured, how long is the result E – D?




2. Use this take-away approach to find the following differences.

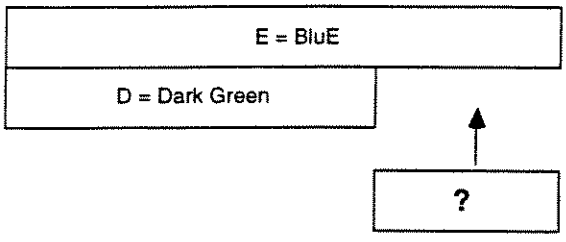
   a. O – Y = b. (O + R) – K = c. (O + P) – D =

3. Marcie lives 13 blocks from the mall. If she has walked 6 blocks toward the mall, how many more blocks must she walk to reach the mall? In this case, we are looking for what needs to be added to 6 to obtain the result of 13. The picture below illustrates how this approach would find E – D. Use your strips to solve the following:

   a. Y + ___ = O b. K + ___ = O + R

   c. D + ___ = O + P

   In these problems, you were looking for the **"missing addend."** How do the addition problems in part 3 relate to the subtraction problems in part 2?




4. Rewrite each of the following subtraction problems into its missing-addend equivalent and find the solution.

   a. N – R = ___ b. K – D = ___ c. (O + W) – Y = ___

5. The table to the right is the one you should have obtained in Activity 3.2. Using the missing-addend approach and this table, find the following differences.

   a. N – Y = _____ b. O – Y = _____

   c. K – P = _____ d. P – R = _____

   Use your centimeter strips to check your results.

| + | W | R | G | P | Y |
|---|---|---|---|---|---|
| W | R | G | P | D | K |
| R | G | P | Y | D | K |
| G | P | Y | D | K | N |
| P | Y | D | K | N | E |
| Y | D | K | N | E | O |

# Mental Math Problems
## (Section 4.1)

(1) $63x97$

(2) $51x212$

(3) $3112 \div 62$

(4) $4254 \div 68$

(5) $347 + 362 + 354 + 336$

(6) $42x38x41$

(7) $198 + 387$

(8) $132 - 96$

(9) $250 - 167$

(10) $3 \cdot 150 + 7 \cdot 150$

(11) $16 \cdot 32 - 6 \cdot 32$

(12) $52 \cdot 14 - 52 \cdot 4$

(13) $3 + 4x5$

(14) $12x2 - 3x5$

(15) $4 + 8 \div 4x2$

(16) $4x13x25$

(17) $37 + 25 + 43$

(18) $5x78x20$

(19) $5(37 \cdot 2)$

(20) $14(19)$

(21) $27(999)$

(22) $3^2 + 4^2$

(23) $(3 + 4)^2$

(24) $2^7 2^3$

18

Name _____ Date _____

# Order of Operations

(Answer ID # 0588953)

**Simplify.**

1. $(512 \div 8) \times 360 \div 6 - (55 - 2) + 5 \times 43$
2. $(791 + 3) + (192 - 2 \times 26) - 25 + 17 \times 20$
3. $69 \times 50 + 124 \times 920 \div 8 - 4 - 482$
4. $(708 \div 3 \times 425 + 5) \div (60 \div 4)$
5. $48 \div 4 - 45 \div 9 + 30 \times (50 \times 984 \div 3)$
6. $(9 - 4) + 265 \times (768 \div 6) \times 13 - 507$
7. $558 + 25 - 159 + 72 \div 6 + 4 \times 720 \div 6$
8. $65 \times 3 - 4 \times 51 + 392 + (696 \div 6) - 185$
9. $(645 \times 276 \times 83 + 16{,}438) \div (574 + 95 \times 317)$
10. $(89 + 4) \times 483 \div 3 + 240 \times 3 \times (52 \div 4)$
11. $290 \div 5 + 1 \times 521 + 66 \div 6 - 391$
12. $126 \div 6 + 536 \div 4 + 32 \times 1 \times 136 - 287$
13. $(64 \div 8) \times 49 + 1 - 5 + 3 - (790 \div 5)$

18

19

# Answer Key 0588953

**1** 4002
**2** 1249
**3** 17224
**4** 6687
**5** 492007
**6** 440458
**7** 916
**8** 314
**9** 482
**10** 24333
**11** 199
**12** 4220
**13** 233

19

**The Trouble With OTIS ...**

Otis had problems.

He didn't seem to fit in too well...

Everybody treats me like I'm nothing!

Sorry, Otis, but we just naturally can't count on you.

$\{1,2,3,...\}=\mathbb{N}$

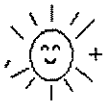
Don't you dare try to come between us, Otis! We're little enough as it is!

.3

Sorry, Otis, but we can't let you in here without the proper sign.

$\{\pm1, \pm2, \pm3, \dots\}$

Otis did finally find his little niche ...

$\{\dots,-3,-2,-1,$  $+1,+2,+3,\dots\}$

But even here he had problems ...

Poor Otis never seems to change. He doesn't have much personality.

9

Yeah, he could even stand on his head and he'd still look the same.

6

Even his girlfriend decided to leave him ...

But, Eunice, I'm nothing without you!

1

That's just it, Otis. On your own, you'll never amount to anything!

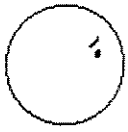
1

And as if that were not enough, Otis had trouble with this set of characters ...

{+, −, X, ÷}

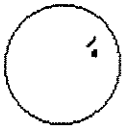
Otis has got to go, Boss!

Yeah, we can't get anywhere with him around!



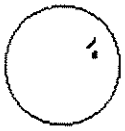
+ − R

I tell you, boss, ever since that guy joined the firm, I've had nothing but setbacks!



X R

Either he goes or I go!



÷ R

Listen, Buster, this town ain't big enough for both of us!

÷

After that last remark, Otis became so frightened that he even tried to change his looks.

But ...

It's all a matter of muscle control, Otis. You'll just have to pinch your tummy in.

8

Poor Otis!

Things went from bad to worse, until one day he happened to overhear this remark ...

Oh, Otis is all right I guess, as long as he stays in his place.

9,246,530

This made Otis so MAD that he told them all off ...

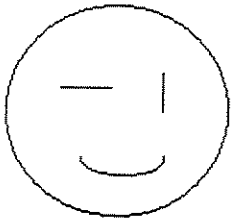
Just remember you guys! If it weren't for me, you wouldn't be in the position you're in now!

9,246,530

... and in doing so, Otis realized his own real worth.

He DID have a place in the real world after all.

THE END



By Mary C. Cornelius

The Triumph of Eunice

or

If You Think Otis had problems, well ...

Ladies and Gentlemen. I'm sure you remember Eunice ...

Skinny, but nice.

You may not think she was so nice after she treated Otis the way she did.

You DO remember Otis, of course ...

Those two hit it off right from the start, it seemed. Some say they got along so well because he was so fat and she was so thin. But their romance was doomed, for reasons you already know ...

But Eunice, I'm nothing without you!

- Flashback -

That's just it, Otis. On your own you'll never amount to anything.

- Flashback -

So, Eunice decided to go it alone. You see, she had one advantage that Otis didn't have.

Eunice was a real natural number and could always be counted on.

$N=\{1,2,3,\ldots\}$

But ...

I'm very sorry, Miss Eunice, but we need exponential employees who will increase production.

$\{x : x^n > x\} = S$

Eunice has got to go, boss!

Yeah, boss, she's a real drag! All she does is slow us up!

$+ \quad - \quad R$

Look, boss, we know she's not as bad as Otis; but with her around we're at a dead standstill!

$\times \quad \div \quad R$

Needless to say, Eunice wasn't very popular. And to make matters worse, the other numerals made fun of her shape. In fact, according to them ...

Tsk, tsk! No shape at all!

89

Nothing but skin and bones!

63

Eunice wasn't completely without friends, however. There was one who shared a similar fate -- a German fellow named Heinz.

1

(Don't let his looks disturb you, Eurpoean ones usually look like this.)

Heinz did his best to boost Eunice's morale ...

Don't let them bug you, Eunice. Look at it this way: We're not underweight; everyone else is just overweight.

1

Unbeknownst to Eunice, however, Heinz entered her in a beauty contest.
Don't laugh, Heinz knew her potential; and no one else -- not even Eunice herself -- was even remotely aware of it.
Eunice eventually found out about the contest, of course; and she was heartbroken. She knew what the others would think ...

But Heinz, they'll all laugh at me! They say I'm as flat as an ironing board!

1

Yes, Eunice could hear it all now. She would be the laughing stock of the entire set ...

Tsk, Tsk! Poor Eunice!

2

It's a shame Otis couldn't give her some of his excess!

3

And of all numerals to enter a beauty contest!

4

Can You Imagine?

5

She sure has got a lot of nerve!

6

Even I have more shape than she does!

7

Like an ironing board, I say!

8

Ha! Beanpole you mean!

9

Finally came the day that Eunice dreaded -- the day of the judging -- but it turned out to be like Judgment Day for all the rest ...
... for would you believe --

Would you believe --

That Eunice Actually Won?

I still don't see why she got first place!

censored

1st 2nd 3rd 4th 5th 6th 7th 8th 9th

Yes, Eunice walked off with top honors; and all the others (all except Heinz, of course) were absolutely green with envy.
Eunice's potential was no longer obscure; and Eunice beamed with pride, for now she too knew that no matter how you looked at it, and no matter who did or didn't like it ...

... Eunice would always be

FIRST

on everybody's list!


THE END

by Mary C. Cornelius

25 See I know thirty
+37 + twenty = fifty
62 seven + five = 12
fifty + 12 = 62.

Student 1

25 62
+37

Student 2

25
+37
62

Student 3

1
25 I added the
+37 5 and the 7
62 together that
is 12 so I
carried the 1
and put down
the 2. 1 2 3 =
6. I put down
the 6 so it
62

Student 4

25 2+3=5 and
+37 5+7=
53 12
So I had 512 I add
two to the one
and made 53

Student 5

25
+37 5+7 is 12
125 is 5 2+3

Student 6

What are they thinking?

$$\begin{array}{r} 23 \\ -15 \\ \hline 12 \end{array}$$

$$\begin{array}{r} {}^{4}5\,{}^{15}6\,{}^{1}2 \\ -\ 2\ 3\ 7 \\ \hline 3\ 2\ 5 \end{array}$$

$$\begin{array}{r} {}^{3}5\,{}^{1}6\,{}^{1}2 \\ 2\ 8\ 7 \\ \hline 1\ 8\ 5 \end{array}$$

$$\begin{array}{r} {}^{1}53 \\ \times\ \ 4 \\ \hline 242 \end{array}$$


24

# Great Books For The Elementary School Library

| | | |
|---|---|---|
| A Cloak For The Dreamer | Aileen Friedman | Shapes |
| A Remainder Of One | Elinor Pinczes | Fractions |
| Amanda Bean's Amazing Dream | Marilyn Burns | Counting |
| Anno's Math Games | Mitsumasa Anno | |
| Each Orange Had 8 Slices | Paul Giganti | Fractions |
| Family Math | Stenmark, Thompson, etc | Math with parents |
| Grandfather Tang's Story | Ann Tampert | Tangrams |
| How Big is a Foot? | Rolf Myller | Measurement |
| How Much Is A Million? | David Schwartz | Large numbers |
| If You Hopped Like A Frog | David Schwartz | ratios |
| If You Made A Million | David Schwartz | Large numbers |
| Knots On A Counting Rope | Bill Martin jr. | Pythagoras |
| Marvelous Math A Book Of Poems | Lee Bennett Hopkins | |
| Math Curse | Joh Sciezzka | Math is a problem |
| Measuring Penny | Loreen Leedy | Measurement |
| Missing Piece | Shel Silverstein | Poems |
| One Grain Of Rice | Demi | Exponential growth |
| One Hundred Hungry Ants | Elinor Pinczes | Counting |
| One Hungry Cat | Joanne Rocklin | |
| Phantom Toll Booth | Norton Jester | MATH |
| Read Any Good Math Lately? | David J. Whiting | Stories behind situations |
| Sir Cumference And The Dragon Of Pi | Cindy Neuschwander | Pi |
| Sir Cumference And The First Round Table | Cindy Neuschwander | Pi |
| Socrates And The Three Little Pigs | Tuyosi Mori | Logic |
| Spaghetti & Meatballs for All | Marilyn Burns | Area, perimeter |
| Storytime Mathtime | Patricia Satariano | Math in unexpected places |
| Ten Little Rabbits | | Counting |
| The Button Box | | Sets |
| The Doorbell Rang | Pat Hutchins | Manipulatives |
| The Dot And The Line | Norton Juster | Geometry |
| The Grapes of Math | Greg Tang | Riddles |
| The Greedy Triangle | Marilyn Burns | Polygons |
| The Hershey's Milk Chocolate Fractions Book | Jerry Pallotta | Fractions |
| The History Of Counting | | Counting |
| The King's Chessboard | | Exponential growth |
| The King's Commissioners | Marilyn Burns | Counting |
| The Librarian Who Measured The Earth | Kathryn Lasby | Measurement |
| The Little Book Of Big Primes | Paul Ribenboim | Primes |
| The M&M Counting Book | | Counting |
| The Teacher From The Black Lagoon | Mike Thaler | Fear of school |
| Two Ways to count to Ten | Ryby Dee | Counting |
| Catalogs to order: | Nasco | 1-800-558-9595 |
| | www.enasco.com | |
| | Creative Publications | 1-800-642-0822 |
| | www.wrightgroup.com | |
| | Dale Seymore | 1-800-872-1100 |
| | www.aw.com/dsp | |

## SELF-TEST

**Multiple Choice:**

1. Which of the following computations can be simplified using distributivity?

   a. $34(17 + 83)$ b. $20(5 \times 163)$ c. $17 + (13 + 42)$ d. $27 \times 7 + 27 \times 3$

2. Which of the following illustrates the left-to-right addition method for finding 35 + 41?

   a. 30 + 40 + 5 + 1
   b. 30 + 5 + 40 + 1
   c. 5 + 1 + 30 + 40
   d. None of a, b, or c.

3. Which of the following are equal?

   (i) $37_{\text{eight}} + 124_{\text{eight}}$ (ii) $354_{\text{eight}} - 152_{\text{eight}}$ (iii) $32_{\text{eight}} \times 5_{\text{eight}}$

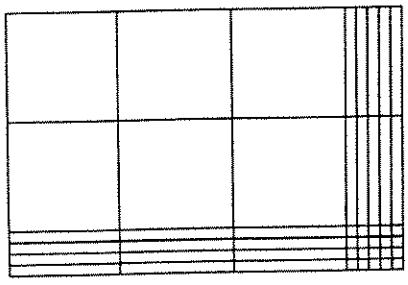
   a. i and ii b. i and iii c. ii and iii d. None of a, b, or c.

**Short Answer:**

4. Show how to estimate the following sum in two different ways.

   640 + 673 + 713 + 702 + 645

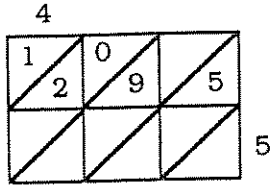
5. What multiplication problem is represented by the given arrangement of pieces and the rectangular array approach? Explain how the arrangement of these pieces relates to the partial products in the intermediate algorithm for multiplication.



6. You are sent to the donut shop with $5.00 to buy 5 dozen donuts for the office staff. You notice that the price is listed as $1.20 per dozen, but that a special is advertised for $.25 off each dozen when you buy 5 dozen or more. Will you be able to buy 5 dozen of the donuts? How much will you pay the salesclerk? Discuss when an estimate would be appropriate in this scenario and when an exact calculation is necessary.

7. Find the indicated product by completing the lattice to the right.

   What factors were multiplied?




8. Explain how you would find the remainder for 6,289,214 divided by 92,365 on a calculator.

**Application/Problem Solving:**

9. a. Given below is a sample of a student's paper. Find the error pattern the student is using and complete the last two items using the same error pattern.

| | | | | |
|---|---|---|---|---|
| $\overset{6}{1}\overset{1}{\cancel{7}}9$ | $\overset{3}{2}\overset{1}{\cancel{4}}7$ | $\overset{4}{2}\overset{1}{\cancel{5}}6$ | 372 | 486 |
| $-\ 26$ | $-\ 25$ | $-\ 39$ | $-\ 48$ | $-\ 72$ |
| 1413 | 2112 | 217 | | |

b. What instructional procedures might you use to help the student with this problem?

10. Instead of dividing by 24 in the problem shown below, a sixth grader decides it would be easier to divide by 4 and then divide that result by 6.

$24\overline{)1579}$

$$\begin{array}{r} 394\ r3 \\ 4\overline{)1579} \\ 12 \\ \hline 37 \\ 36 \\ \hline 19 \\ 16 \\ \hline 3 \end{array}$$

$$\begin{array}{r} 65\ r4 \\ 6\overline{)394} \\ 36 \\ \hline 34 \\ 30 \\ \hline 4 \end{array}$$

a. Does this process give you the correct quotient? the correct remainder?

b. The divisor 24 can also be written as $3 \times 8$. What results do you get using these numbers?

c. Explain what is happening in this process. How can you obtain the correct results?

11 16. Compute the following without using your abacus:

a. $1221_{three} + 2122_{three}$

b. $2211_{three} - 1212_{three}$

a. $12_{three} \times 111_{three} =$ _____

b. $21_{three} \times 21_{three} =$ _____

c. $21_{three} \times 201_{three} =$ _____

d. $12_{three} \times 120_{three} =$ _____

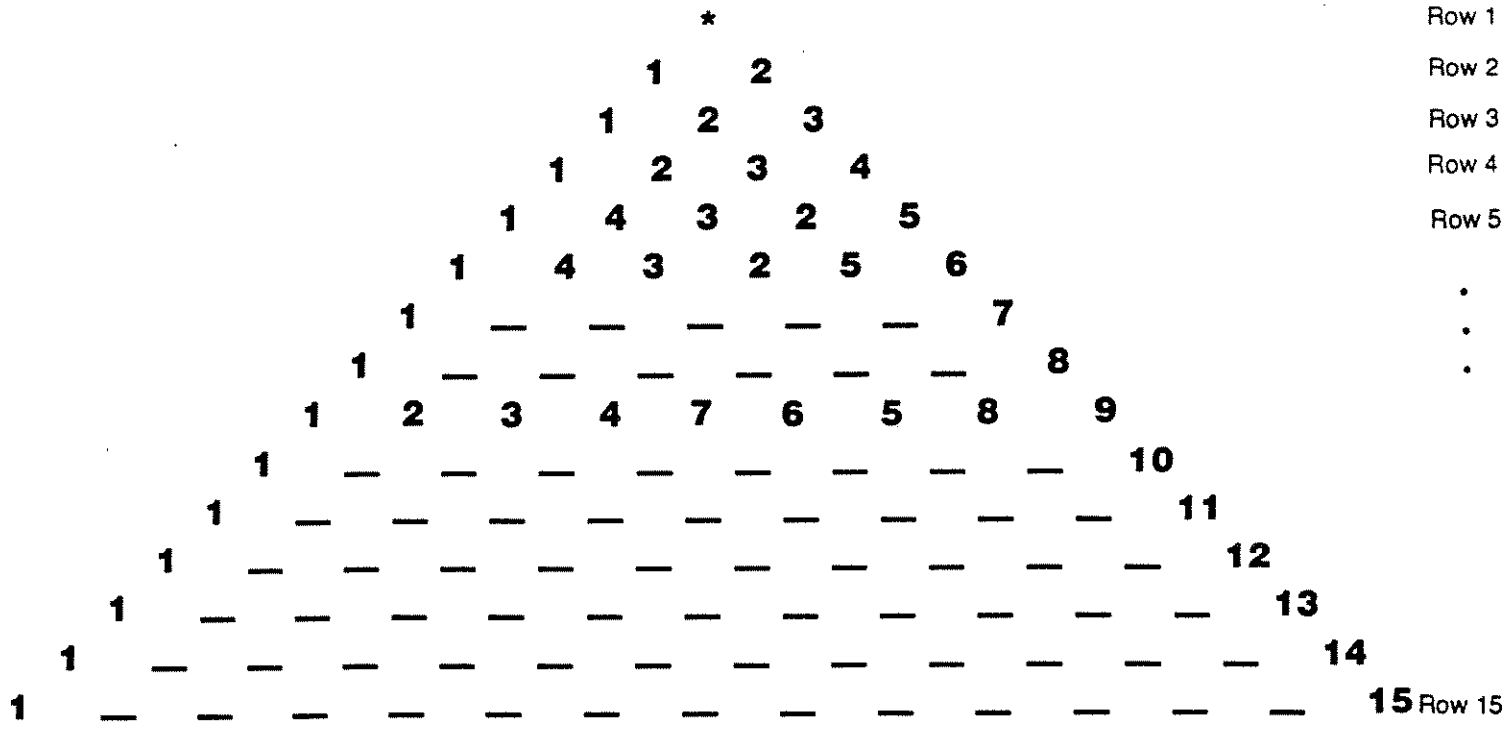
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
STUDENT math notes
NOVEMBER 1986

# Primes

A *prime number* is a natural number that has exactly two factors, itself and 1. The pyramid below is called a *prime pyramid*. Each row in the pyramid begins with 1 and ends with the number that is the row number. In each row, the consecutive numbers from 1 to the row number are arranged so that the sum of any two adjacent numbers is a prime.

For example, look at row 5:

1) It must contain the numbers 1, 2, 3, 4, and 5.
2) It must begin with 1 and end with 5.
3) The sum of adjacent pairs must be a prime number.
4) 1 + 4 = 5, 4 + 3 = 7, 3 + 2 = 5, and 2 + 5 = 7.




- Supply the missing numbers in this prime pyramid.
- Can you extend the prime pyramid beyond row 15?
- What patterns do you see in your solutions?
- What is your solution strategy for completing the pyramid?

The editors wish to thank Margaret Kenney, Mathematics Institute, Boston College, Chestnut Hill, MA 02167, for writing this issue of the *NCTM Student Math Notes*.



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# Prime Start

Euclid (born about 300 B.C.), a Greek mathematician famous for his geometry, showed that the number of primes is infinite. Eratosthenes (born in 230 B.C.), also a Greek, was the chief librarian at the University of Alexandria in Egypt. He developed a method for finding primes that is still in use today.

## Sieve of Eratosthenes

Start with a 10 × 10 array of the numbers 1 through 100. The number 1 is crossed out because it is not a prime. Why not? _____

Circle 2, the first prime; then cross out the remaining multiples of 2 using a horizontal slash (—).

What visual pattern do the multiples of 2 make in the grid? _____

What's the next number that hasn't been crossed out? _____

Circle 3, the next prime; then cross out the remaining multiples of 3 using a vertical slash ( | ).

What visual pattern is made by the multiples of 3? _____

What is the next prime number that hasn't been crossed out? _____

Circle the 5 and cross out the remaining multiples of 5 using the diagonal slash ( / ).

In a similar manner, circle the next prime, 7, and cross out all its remaining multiples using the reverse diagonal slash ( \ ).

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

At this point, except for 2, 3, 5, and 7, you have sifted out all the multiples of 2, 3, 5, and 7.

1. Are any multiples of 11, the next prime, not yet crossed out? _____

2. Why is this the case? _____

_____

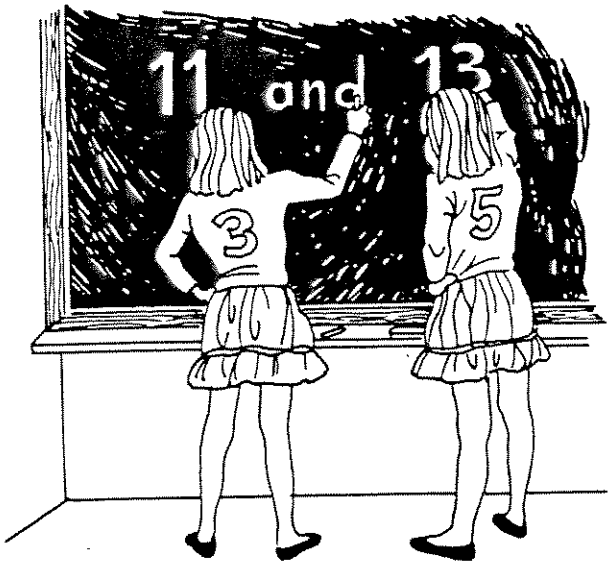
The numbers that are not crossed out are the primes less than 100. Make a list of these primes.

2, 3, 5, 7, 11, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, ____, and ____.

# Twin Primes

Several pairs of primes in the list of primes less than 100 have a difference of 2. For example, the pairs 3 and 5. 5 and 7, and 11 and 13 each have a difference of 2. These pairs are called *twin primes*. Complete the list of all twin primes less than 100. Also, find the sums and products of these twin primes.

| Twin Primes | Sums | Products |
|---|---|---|
| 3 and 5 | _____ | _____ |
| 5 and 7 | _____ | _____ |
| 11 and 13 | _____ | _____ |
| ____ and ____ | _____ | _____ |
| ____ and ____ | _____ | _____ |
| ____ and ____ | _____ | _____ |
| ____ and ____ | _____ | _____ |
| ____ and ____ | _____ | _____ |




1. Do you see a pattern in the column of sums? Can you *prove* a fact about the *sums* of twin primes?

2. Do you see a pattern in the column of products? Can you *prove* a fact about the *products* of twin primes?

3. Examine the primes larger than 5 in your list. What *different* digits appear in the units position of these primes? _____ Will any prime larger than 100 have a different ending than the ones you have found? _____

The number 13 is a prime, and 31, the reverse of 13, is also a prime. 13 is called an *emirp* (*prime* spelled backward) because its reverse is a *different prime*. 31 is also an emirp. But 11 is not an emirp. Why not? _____

4. List all the emirps less than 100. _____

# Prime Concerns

Over the centuries we have learned a great deal about prime numbers. But in all this time no one has discovered a simple formula that will produce all the primes starting with 2. Many attempts have been made, and no doubt will continue to be made, to find such a formula. One such attempt produced the following:

$$p_n = n^2 - n + 41$$

$p_n$ is a prime number for $n = 1$ through $n = 40$. For example, $p_1 = 41$, $p_2 = 43$, and $p_3 = 47$. Use your calculator or write a computer program to find additional values of $p_n$ for $n = 4, \ldots, 40$. What is $p_{41}$? _____ Is it prime or composite? Why? _____

5. What are some other values of $n > 41$ for which $p_n$ is a composite number? _____

6. Is $p_n$ prime for some values of $n > 41$? If so, list some of them. _____

## Can You . . .

- replace each blank with a "+" or "−" to get an equality relation?

$17 = 1 \_\_\_ 2 \_\_\_ 3 \_\_\_ 5 \_\_\_ 7 \_\_\_ 11 \_\_\_ 13 \_\_\_ 13$

$19 = 1 \_\_\_ 2 \_\_\_ 3 \_\_\_ 5 \_\_\_ 7 \_\_\_ 11 \_\_\_ 13 \_\_\_ 17$

$23 = 1 \_\_\_ 2 \_\_\_ 3 \_\_\_ 5 \_\_\_ 7 \_\_\_ 11 \_\_\_ 13 \_\_\_ 17 \_\_\_ 19 \_\_\_ 19$

$29 = 1 \_\_\_ 2 \_\_\_ 3 \_\_\_ 5 \_\_\_ 7 \_\_\_ 11 \_\_\_ 13 \_\_\_ 17 \_\_\_ 19 \_\_\_ 23$

- find some emirps that contain three, four, or more digits?
- in Eratosthenes' sieve, name the primes whose multiples *must* be crossed out to find all primes less than 200? 300? 500?
- write a computer program based on Eratosthenes' sieve to find all primes less than 1000?
- find a string of at least 1 million consecutive numbers that are all composite? For example, 24, 25, 26, 27, 28 is a string of five consecutive numbers that are all composite. Identify your string by naming the first number and the one-millionth number in the string. (*Hint:* Use factorials!)

## Did You Know That . . .

- a conjecture states that the number of twin primes is infinite? No one has been able to prove or disprove this conjecture.
- J. P. Kulik spent twenty years, unassisted, computing a factor table of the numbers from 1 to 100 000 000? He completed his monumental work in 1867. It filled eight volumes, but volume 2 is now missing from the collection at the Vienna Royal Academy.
- a special kind of prime number, *Mersenne numbers,* are named after the French priest and amateur mathematician Marin Mersenne? In 1644, he published his (incomplete) list of primes that satisfied the rule $M_p = 2^p - 1$, where $p$ is prime: $p = 2, 3, 5, 7, 13, 17, 19, 31, 67, 127, 257$. It took 304 years to resolve the errors in his short list.
- in 1978, the twenty-fifth Mersenne prime, $2^{21\,701} - 1$, was found by two eighteen-year-old students, Laura Nickel and Curt Noll, using a CYBER-174 computer at California State University at Hayward?
- the largest known Mersenne prime, $2^{216\,091} - 1$, consists of 65 050 digits and was discovered in 1985 in Houston, Texas, on a Cray X-MP supercomputer by scientists at Chevron Geosciences Company? This find will probably be declared the thirtieth Mersenne prime.
- Christian Goldbach, 1690–1764, conjectured that every even number greater than 4 is equal to the sum of two prime numbers? His conjecture remains unproved today.

**NCTM STUDENT MATH NOTES** is published as part of the **NEWS BULLETIN** by the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091. The five issues a year appear in September, November, January, March, and May. Pages may be reproduced for classroom use without permission.

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Printed in U.S.A.

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# Teacher Notes

**Page 7.** The prime-number pyramid offers an interesting problem-solving activity. Be certain students understand the conditions and requirements. It may be helpful for some to begin with an array where the numbers are listed in order in each row and then rearranged as needed so that successive pairs have prime sums. Note from the example shown that the prime sums can be the same for different pairs in any given row.

**Page 8.** Encourage students to do some reading and research on the lives of the Greek mathematicians Euclid and Eratosthenes. Extending the sieve to 1 through 200 would require continuing the process through only the next two primes, 11 and 13.

**Page 9.** Many interesting properties of prime numbers have been found. The formula $P_n = n^2 - n + 41$ was first introduced by Euler in 1772. Several others are given following the answers below.

**Page 10.** A Mersenne prime is a prime number in the form $2^p - 1$, where $p$ is prime. In 1992, another Mersenne prime was discovered. This new prime, found using a Cray-2 supercomputer, is $2^{756839} - 1$. The magnitude of this prime number is well reflected by the fact that it contains 227 839 digits. The previous record-holding Mersenne prime, found in 1985, was $2^{216091} - 1$ and consisted of a mere 65 050 digits.

# Answers

**Page 7.**

1 4 3 2 5 6 7
1 2 3 4 7 6 5 8
1 2 3 4 7 6 5 8 9
1 2 3 4 7 6 5 8 9 10
1 4 3 2 5 6 7 10 9 8 11
1 4 3 2 5 6 7 10 9 8 11 12
1 4 3 2 5 6 7 12 11 8 9 10 13
1 2 5 8 3 4 7 12 11 6 13 10 9 14
1 2 5 8 3 4 7 12 11 6 13 10 9 14 15

**Page 8.** **1.** No **2.** You do not need to cross out the multiples of 11, since the first one not already crossed out is $11 \times 11 = 121$, which is greater than 100.

2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, and 97

**Page 9.** Twin primes: 3 and 5, 5 and 7, 11 and 13, 17 and 19, 29 and 31, 41 and 43, 59 and 61, 71 and 73

**1.** Except for 3 + 5, all are divisible by 12: $(6m - 1) + (6m + 1) = 12m$.

**2.** Each product is 1 less than the square of the average of the twin primes.

**3.** 1, 3, 7, 9; No

**4.** 13, 17, 31, 37, 71, 73, 79, and 97

**5.** $n$ = 42, 45, 50, 57, 66, 77, 82, 83, 85, 88, 90, 92, 97

Only thirteen of the first hundred values of $n$ in the formula do not produce primes.

**Page 10.**

- + − − + − + +
  + − + + + + −
  − + + − − + − + +
  + − − + − + − + +

- Emirps: 337, 733, 1471, 1741, ...
- 2, 3, 5, 7, 11, 13; 2, 3, 5, 7, 11, 13, 17; 2, 3, 5, 7, 11, 13, 17, 19
- Here is a string of 1 million consecutive numbers that are all composite:
  $(10^6 + 1)! + 2, (10^6 + 1)! + 3, (10^6 + 1)! + 4, ..., (10^6 + 1)! + (10^6 + 1)$

For some other explorations with primes, try these:

- List numbers of the form $6a \pm 1$, where $a$ = 1, 2, 3, 4, 5, ...

| $a$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $6a \pm 1$ | 5 7 | 11 13 | 17 19 | 23 25 | 29 31 | 35 37 |

  Every prime number greater than 3 will appear in this list, but not every number listed will be prime.

- List every number in the form $(2m - 1)(1 + 2a)$, where $a$ = 1, 2, 3, 4, 5, ... and $m$ = 2, 3, 4, 5, 6, .... Combine the numbers with all even numbers greater than 2 to get all successive nonprime numbers. The omitted odd numbers greater than 1 form, progressively, all the primes.

| | | | | | | | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nonprimes | | | 4 | | 6 | | 8 | 9 | 10 | | 12 | | 14 | 15 | 16 | | 18 | | 20 | 21 | 22 | | 24 | 25 |
| Primes | 2 | 3 | | 5 | | 7 | | | | 11 | | 13 | | | | 17 | | 19 | | | | 23 | | |

**Page 12 (Extension).** **1.** 9; 36% **2.** 30.0%, 25.0%, 23.0%, 20.6%, 19.5%, 19.0%

**4.** As $N$ increases, the percent of prime numbers decreases, asymptotically, toward 0.

For $N$ = 1000, $P$ = 16.8%. For $N$ = 10 000, $P$ = 12.3%. For $N$ = 100 000, $P$ = 9.6%.

# Extension—THE FIRST 500 PRIMES

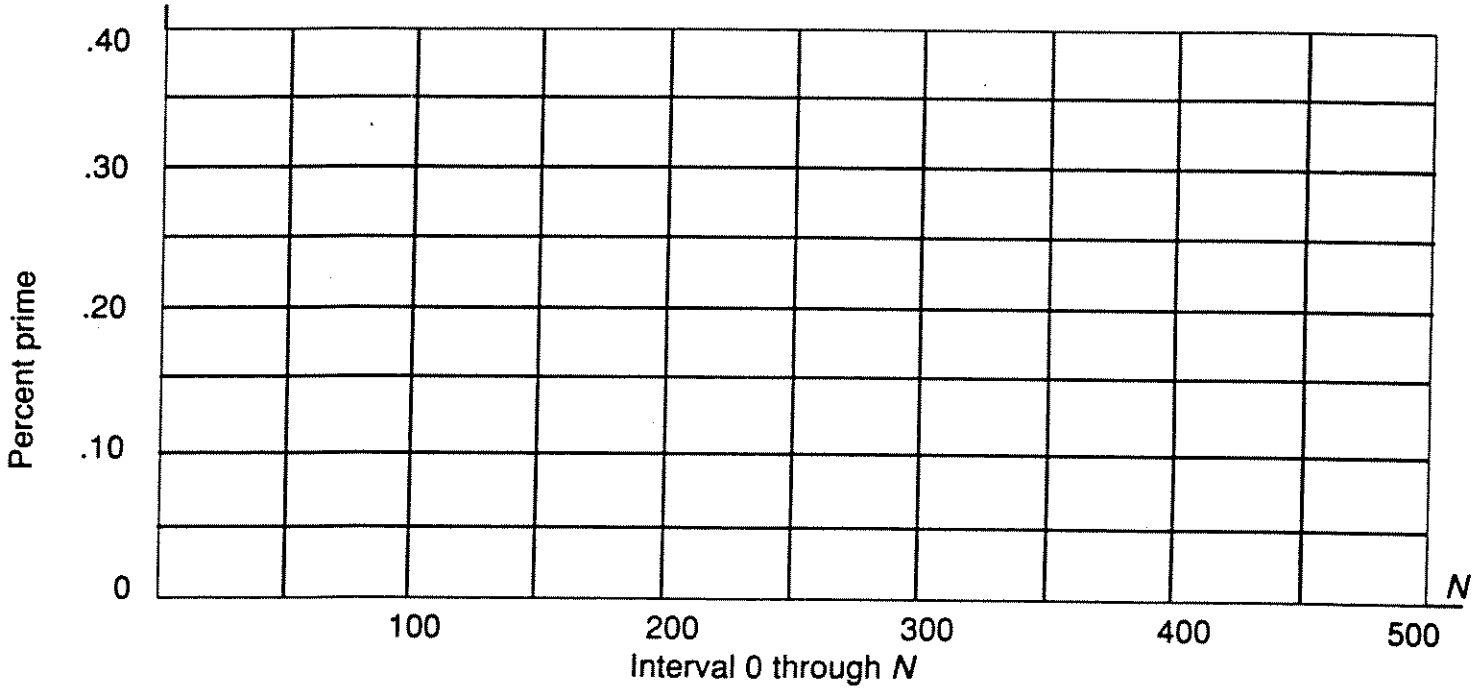
| | | | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | 101 | 233 | 383 | 547 | 701 | 877 | 1049 | 1229 | 1429 | 1597 | 1783 | 1993 | 2161 | 2371 | 2579 | 2749 | 2957 | 3187 | 3373 |
| 3 | 103 | 239 | 389 | 557 | 709 | 881 | 1051 | 1231 | 1433 | 1601 | 1787 | 1997 | 2179 | 2377 | 2591 | 2753 | 2963 | 3191 | 3389 |
| 5 | 107 | 241 | 397 | 563 | 719 | 883 | 1061 | 1237 | 1439 | 1607 | 1789 | 1999 | 2203 | 2381 | 2593 | 2767 | 2969 | 3203 | 3391 |
| 7 | 109 | 251 | 401 | 569 | 727 | 887 | 1063 | 1249 | 1447 | 1609 | 1801 | 2003 | 2207 | 2383 | 2609 | 2777 | 2971 | 3209 | 3407 |
| 11 | 113 | 257 | 409 | 571 | 733 | 907 | 1069 | 1259 | 1451 | 1613 | 1811 | 2011 | 2213 | 2389 | 2617 | 2789 | 2999 | 3217 | 3413 |
| 13 | 127 | 263 | 419 | 577 | 739 | 911 | 1087 | 1277 | 1453 | 1619 | 1823 | 2017 | 2221 | 2393 | 2621 | 2791 | 3001 | 3221 | 3433 |
| 17 | 131 | 269 | 421 | 587 | 743 | 919 | 1091 | 1279 | 1459 | 1621 | 1831 | 2027 | 2237 | 2399 | 2633 | 2797 | 3011 | 3229 | 3449 |
| 19 | 137 | 271 | 431 | 593 | 751 | 929 | 1093 | 1283 | 1471 | 1627 | 1847 | 2029 | 2239 | 2411 | 2647 | 2801 | 3019 | 3251 | 3457 |
| 23 | 139 | 277 | 433 | 599 | 757 | 937 | 1097 | 1289 | 1481 | 1637 | 1861 | 2039 | 2243 | 2417 | 2657 | 2803 | 3023 | 3253 | 3461 |
| 29 | 149 | 281 | 439 | 601 | 761 | 941 | 1103 | 1291 | 1483 | 1657 | 1867 | 2053 | 2251 | 2423 | 2659 | 2819 | 3037 | 3257 | 3463 |
| 31 | 151 | 283 | 443 | 607 | 769 | 947 | 1109 | 1297 | 1487 | 1663 | 1871 | 2063 | 2267 | 2437 | 2663 | 2833 | 3041 | 3259 | 3467 |
| 37 | 157 | 293 | 449 | 613 | 773 | 953 | 1117 | 1301 | 1489 | 1667 | 1873 | 2069 | 2269 | 2441 | 2671 | 2837 | 3049 | 3271 | 3469 |
| 41 | 163 | 307 | 457 | 617 | 787 | 967 | 1123 | 1303 | 1493 | 1669 | 1877 | 2081 | 2273 | 2447 | 2677 | 2843 | 3061 | 3299 | 3491 |
| 43 | 167 | 311 | 461 | 619 | 797 | 971 | 1129 | 1307 | 1499 | 1693 | 1879 | 2083 | 2281 | 2459 | 2683 | 2851 | 3067 | 3301 | 3499 |
| 47 | 173 | 313 | 463 | 631 | 809 | 977 | 1151 | 1319 | 1511 | 1697 | 1889 | 2087 | 2287 | 2467 | 2687 | 2857 | 3079 | 3307 | 3511 |
| 53 | 179 | 317 | 467 | 641 | 811 | 983 | 1153 | 1321 | 1523 | 1699 | 1901 | 2089 | 2293 | 2473 | 2689 | 2861 | 3083 | 3313 | 3517 |
| 59 | 181 | 331 | 479 | 643 | 821 | 991 | 1163 | 1327 | 1531 | 1709 | 1907 | 2099 | 2297 | 2477 | 2693 | 2879 | 3089 | 3319 | 3527 |
| 61 | 191 | 337 | 487 | 647 | 823 | 997 | 1171 | 1361 | 1543 | 1721 | 1913 | 2111 | 2309 | 2503 | 2699 | 2887 | 3109 | 3323 | 3529 |
| 67 | 193 | 347 | 491 | 653 | 827 | 1009 | 1181 | 1367 | 1549 | 1723 | 1931 | 2113 | 2311 | 2521 | 2707 | 2897 | 3119 | 3329 | 3533 |
| 71 | 197 | 349 | 499 | 659 | 829 | 1013 | 1187 | 1373 | 1553 | 1733 | 1933 | 2129 | 2333 | 2531 | 2711 | 2903 | 3121 | 3331 | 3539 |
| 73 | 199 | 353 | 503 | 661 | 839 | 1019 | 1193 | 1381 | 1559 | 1741 | 1949 | 2131 | 2339 | 2539 | 2713 | 2909 | 3137 | 3343 | 3541 |
| 79 | 211 | 359 | 509 | 673 | 853 | 1021 | 1201 | 1399 | 1567 | 1747 | 1951 | 2137 | 2341 | 2543 | 2719 | 2917 | 3163 | 3347 | 3547 |
| 83 | 223 | 367 | 521 | 677 | 857 | 1031 | 1213 | 1409 | 1571 | 1753 | 1973 | 2141 | 2347 | 2549 | 2729 | 2927 | 3167 | 3359 | 3557 |
| 89 | 227 | 373 | 523 | 683 | 859 | 1033 | 1217 | 1423 | 1579 | 1759 | 1979 | 2143 | 2351 | 2551 | 2731 | 2939 | 3169 | 3361 | 3559 |
| 97 | 229 | 379 | 541 | 691 | 863 | 1039 | 1223 | 1427 | 1583 | 1777 | 1987 | 2153 | 2357 | 2557 | 2741 | 2953 | 3181 | 3371 | 3571 |

1. How many numbers from 1 through 25 are prime?
   Express the answer as a percent.

2. Use the table of primes. Find the percent of the numbers in each interval that are prime.

| Interval | 1–50 | 1–100 | 1–150 | 1–200 | 1–250 | 1–300 | 1–350 | 1–400 | 1–450 | 1–500 |
|---|---|---|---|---|---|---|---|---|---|---|
| Percent prime | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ | ____ |

3. Draw a graph of the data collected in question 2.




4. As the number *N* increases, what appears to happen to the percent of prime numbers in the interval from 1 to *N*?

GCF or LCM?

Make a sketch which will help you answer each of these questions. Determine the answer and whether it is the GCF or LCM of the two numbers.

1. Pencils come in packages of 18; erasers that fit on top of these pencils come in packages of 24. What is the smallest number of pencils and erasers that you can buy so each pencil can be matched with an eraser? How many packages of each will you need?

2. Ko has a bag with 45 red candies and another with 75 green candies. She wants to make goody bags so that each goody bag contains the same number of red candies and each goody bag contains the same number of green candies and so that she uses up all of the candies. What is the largest number of goody bags she can make this way? How many of each color will be in the bag?

3. Sam has lots of 8-in sticks he is placing end ot end to make a line of sticks. Becky has 12-inch sticks that she is placing ene to end to make a line of sticks. If they want their line of sticks to be the same length, how long could they be? What is the shortest such length?

4. If I have a room which is 45 by 36 feet, what is the largest square with which I can tile it?

5. If my tiles are 24 by 36 cm. What is the smallest square room I can tile using the tiles in one direction only and not cutting the tiles?

6. a) I have red lights which are 50 to the string and blue lights which are 30 to the string. To hang them from the house, I must put a hanger at the beginning and the end of each strand and evenly space the rest. How often must I put the hangers so I can string the two colors together using the smallest number of hangers?

b) How often will I have a common plug? (that is, reach an end of both colors at the same time?

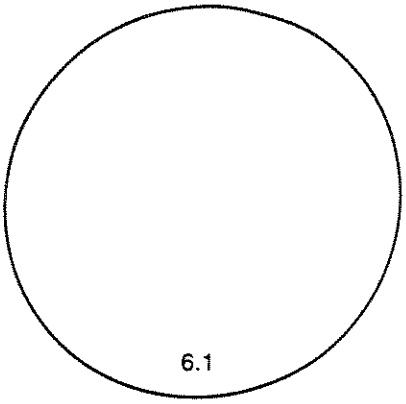
7. a) Red chips come 75 to the package and blue ones come 50 to the package. How do I package them in single color packages so the packages will be as tall as possible, but both types will have the same height (same number of chips in each stack).

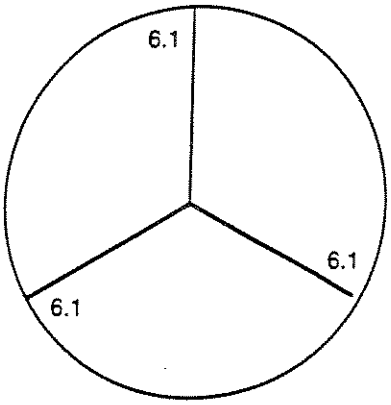
b) If these packages are then put in boxes of single color chips, but I want the same number of chips in each box, what is the least number of chips I can have per box?

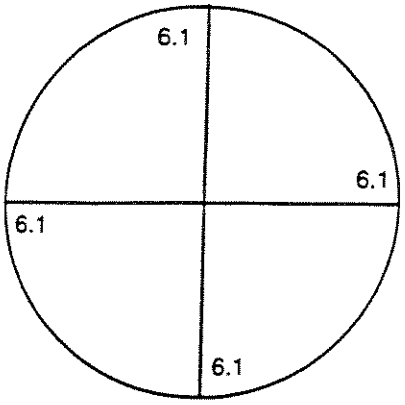
8. Shane, Emily, Jane and Brad play in a string quartet. Their quartet has a 'gig' at a wedding coming up and they need to get together to practice. Emily can only practice with them every 6 days. Shane and Brad can both practice every 4 days. And Jane, who is very busy, can only practice with them every 12 days. Assuming that they can practice together today, when will be the next time they can all get together to practice?

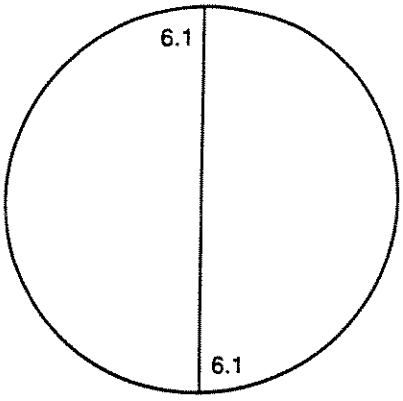
## MATERIALS CARD 6.1

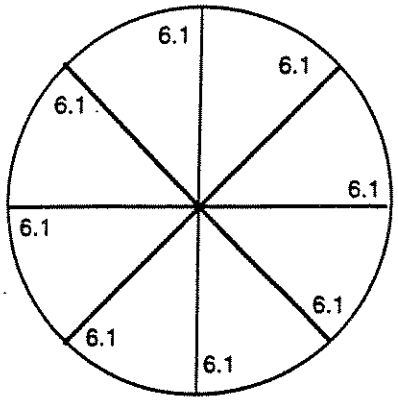
Remove the circles and then cut on all solid lines.
Save these pieces for use later in the chapter.

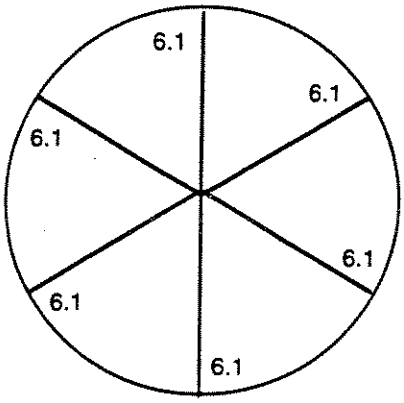


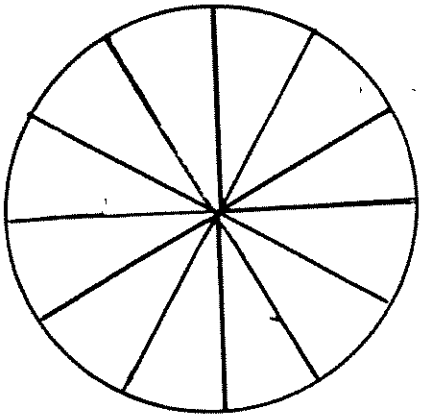



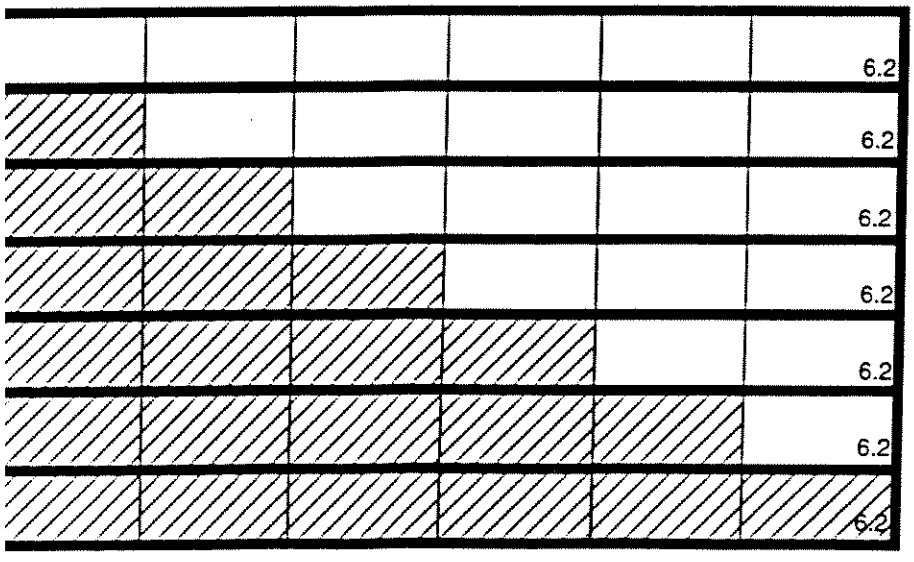



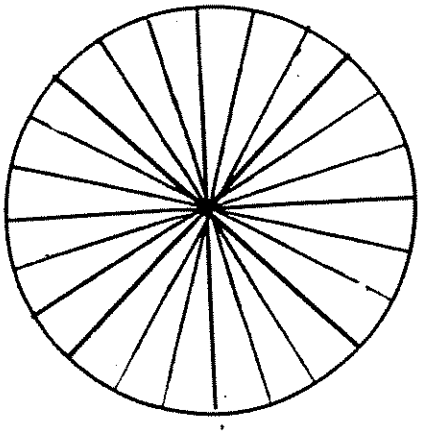



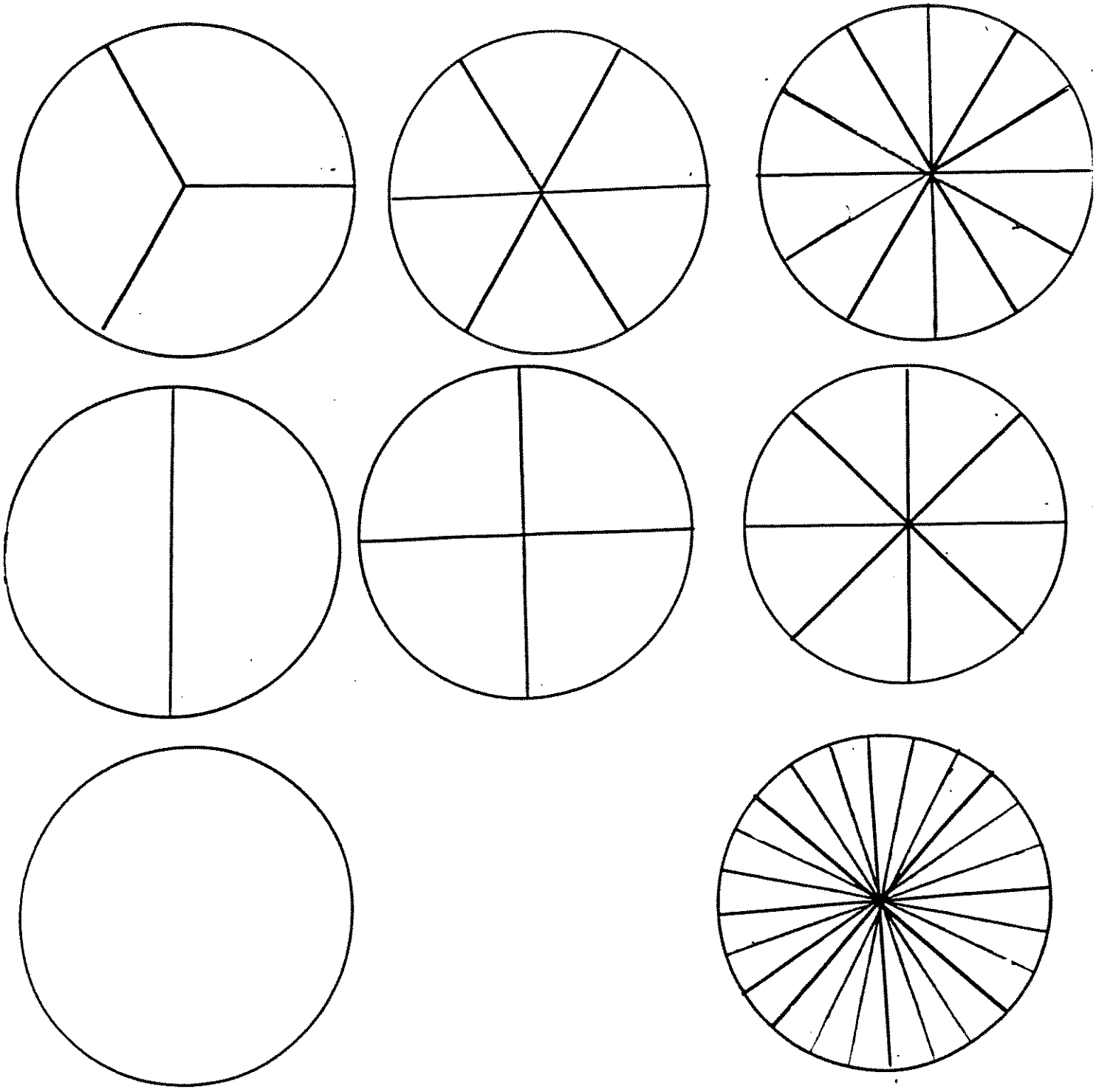





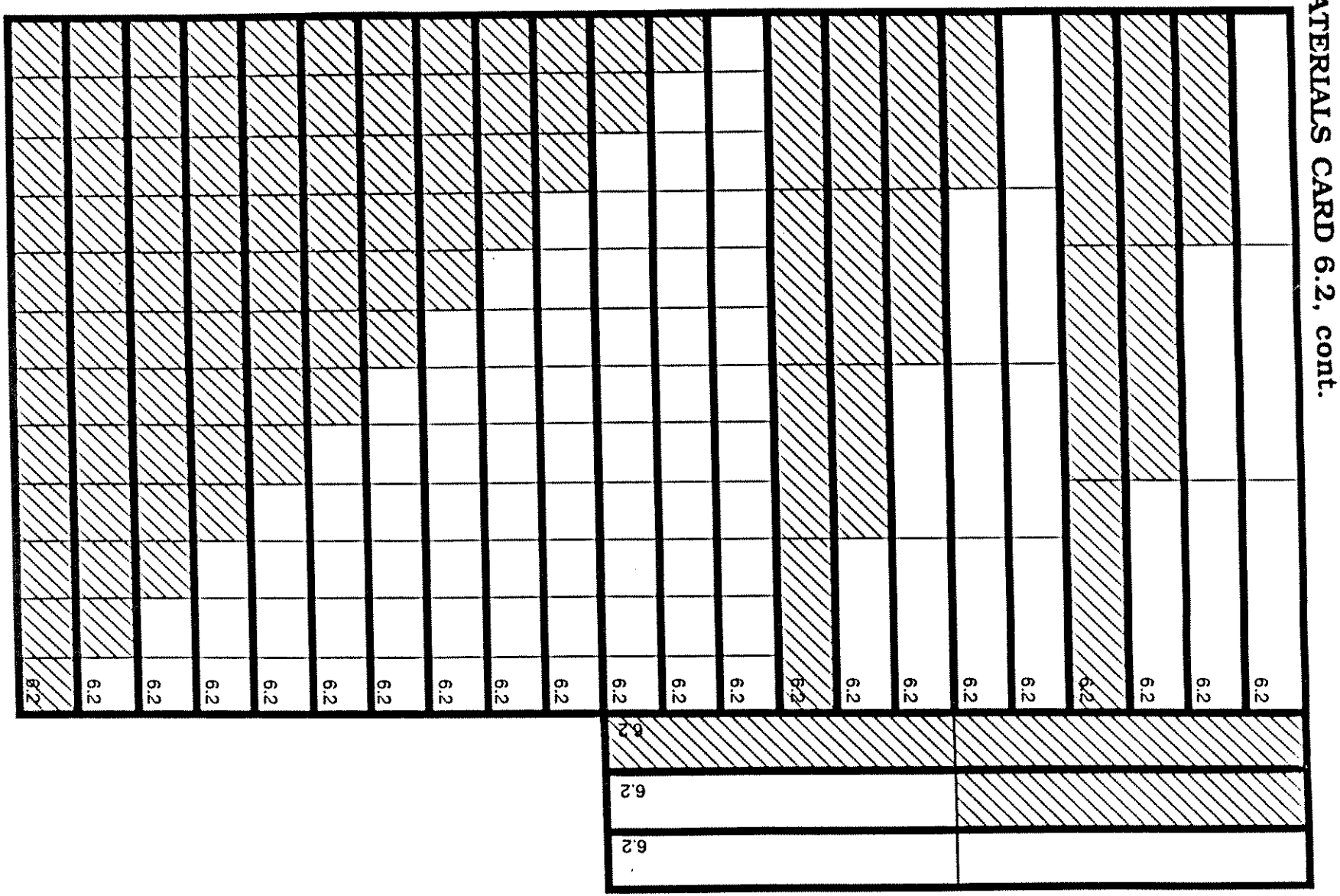



6.1



# MATERIALS CARD 6.2

Remove the fraction strips on this and the next card. Separate them at the heavy dark lines.
Save these strips for use later in the chapter.




MATERIALS CARD 6.2, cont.