Chp 0 (Review)

Rational Exponents

(1) $\sqrt[n]{a} = b \iff a = b^n$ (principal n^{th} root)

(2) $a^{1/n} = \sqrt[n]{a}$

(3) $a^{m/n} = \sqrt[n]{a^m} = (\sqrt[n]{a})^m$

Ex 3 Simplify

(a) $(-8)^{-2/3} = \dfrac{1}{(-8)^{2/3}} = \dfrac{1}{\sqrt[3]{(-8)^2}} = \dfrac{1}{\sqrt[3]{64}} = \dfrac{1}{4}$

(b) $(x^{-2/3})^{-2/5} = x^{\frac{-2}{3}\cdot\frac{-2}{5}} = x^{\frac{4}{15}}$ or $\sqrt[15]{x^4}$

(c) $x^{-2}\cdot x^{5/3} = x^{-2+\frac{5}{3}} = x^{-1/3} = \dfrac{1}{x^{1/3}}$ or $\dfrac{1}{\sqrt[3]{x}}$

(d) $\sqrt{32x^5y} = \sqrt{2^5x^5y} = (2^5x^5y)^{1/2}$

$= 2^{5/2}x^{5/2}y^{1/2} = 2^2 2^{1/2} x^2 x^{1/2} y^{1/2}$

$= 4x^2\sqrt{2xy}$

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Ex 4 simplify.

(a) $\sqrt[3]{16x^2y}\ \sqrt[3]{3x^2y} = \sqrt[3]{\cancel{8}\cdot 2\cdot 3\, x^3 x\, y^2}$

$= 2x\sqrt[3]{6xy^2}$

(b) $\dfrac{\sqrt[3]{-16x^3y^4}}{\sqrt[3]{128y^2}} = \sqrt[3]{\dfrac{-16x^3y^4}{128y^2}} = \sqrt[3]{\dfrac{-x^3y^2}{8}}$

$= \dfrac{-x\sqrt[3]{y^2}}{2}$

(c) $\dfrac{\sqrt[4]{mx^3}}{\sqrt[4]{y^2w^5}}$ (rationalize denominator)

$= \dfrac{\sqrt[4]{mx^3}}{\sqrt[4]{y^2w^4w}} = \dfrac{\sqrt[4]{mx^3}}{w\sqrt[4]{y^2w}}\left(\dfrac{\sqrt[4]{y^2w^3}}{\sqrt[4]{y^2w^3}}\right) = \dfrac{\sqrt[4]{mx^3y^2w^3}}{wyw}$

$= \dfrac{\sqrt[4]{mx^3y^2w^3}}{yw^2}$

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Polynomials $\quad a_n x^n + a_{n-1}x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$

$a_i \in \mathbb{R}$

n = degree

a_i are all coefficients

a_n = leading coefficient

Ex 5 Simplify

(a) $2(x^3+3)(2x^3-5)$

$= 2(2x^6 - 5x^3 + 6x^3 - 15)$

$= 4x^6 - 10x^3 + 12x^3 - 30$

$= 4x^6 + 2x^3 - 30$

(b) $(16x^2 + 4xy^2 + 8x) \div (4xy)$

$= \dfrac{16x^2 + 4xy^2 + 8x}{4xy} = \dfrac{\cancel{4x}(4x + y^2 + 2)}{\cancel{4x}y}$

$= \dfrac{4x + y^2 + 2}{y}$

(c) $(x^4 + 3x^3 - x + 1) \div (x^2+1) \quad = x^2 + 3x - 1 + \dfrac{-4x+2}{x^2+1}$

$$
\begin{array}{r}
x^2 + 3x - 1 \\
x^2+1 \,\overline{\big)\, x^4 + 3x^3 + 0x^2 - x + 1} \\
-(x^4 + x^2) \\
\hline
3x^3 - x^2 - x + 1 \\
-(3x^3 + 3x) \\
\hline
-x^2 - 4x + 1 \\
-(-x^2 - 1) \\
\hline
-4x + 2
\end{array}
$$

Chp 0 (Review)

Ex 6 Factor completely.

(a) $x^2+6x+8 = (x+2)(x+4)$

(b) $4x^2-8x-60 = 4(x^2-2x-15)$
$= 4(x+3)(x-5)$

(c) $x^4-3x^2-4 = (x^2+1)(x^2-4)$
$= (x^2+1)(x-2)(x+2)$

(d) $x^3+6x^2+12x+8 = (x^3+8)+(6x^2+12x)$
$= (x+2)(x^2-2x+4) + 6x(x+2)$
$= (x+2)(x^2-2x+4+6x)$
$= (x+2)(x^2+4x+4)$
$= (x+2)(x+2)(x+2) = (x+2)^3$

Chp 0 (Review)

Fractions (Polynomial)

Ex 7 Simplify.

(a) $\dfrac{x^2y^2-4x^3y}{x^2y-2x^2y^2} = \dfrac{x^2y\,(y-4x)}{x^2y\,(1-2y)} = \dfrac{y-4x}{1-2y}$

(b) $\dfrac{x^2-5x-6}{x^2-5x+4}\cdot\dfrac{x^2-x-12}{x^3-6x^2}\cdot\dfrac{x-x^3}{x^2-2x+1}$

$= \dfrac{\cancel{(x-6)}(x+1)\cancel{(x-4)}(x+3)\,x(1-x^2)}{\cancel{(x-4)}(x-1)\,x^2\cancel{(x-6)}(x-1)(x-1)} = \dfrac{\cancel{x}(x+1)(x+3)(1-x)(1+x)}{(x-1)x^2(x-1)(x-1)}$

(note: $1-x$
$= -(x-1)$)

$= \dfrac{-(x+1)(x+3)\cancel{(x-1)}(x+1)}{x(x-1)(x-1)\cancel{(x-1)}}$

$= -\dfrac{(x+3)(x+1)^2}{x(x-1)^2}$

(c) $\dfrac{3x^2(x+1)}{\sqrt{x^3+1}}+\sqrt{x^3+1} = \dfrac{3x^2(x+1)}{\sqrt{x^3+1}}+\sqrt{x^3+1}\left(\dfrac{\sqrt{x^3+1}}{\sqrt{x^3+1}}\right)$

$= \dfrac{3x^2(x+1)+x^3+1}{\sqrt{x^3+1}} = \dfrac{3x^3+3x^2+x^3+1}{\sqrt{x^3+1}} = \dfrac{4x^3+3x^2+1}{\sqrt{x^3+1}}$

Chp 0 (Review)

EX 8 Simplify.

(a) $$\frac{1-\frac{2}{x-2}}{x-6+\frac{10}{x+1}} = \left(\frac{1-\frac{2}{x-2}}{x-6+\frac{10}{x+1}}\right)\left(\frac{(x-2)(x+1)}{(x-2)(x+1)}\right)$$

LCD = (x-2)(x+1)

$$= \frac{(x-2)(x+1)-2(x+1)}{(x-6)(x-2)(x+1)+10(x-2)} = \frac{x^2-x-2-2x-2}{x^3-7x^2+4x+12+10x-20}$$

$$= \frac{x^2-3x-4}{x^3-7x^2+14x-8} = \frac{\cancel{(x-4)}(x+1)}{(x-1)(x-2)\cancel{(x-4)}} = \frac{x+1}{(x-1)(x-2)}$$

(b) $$\frac{x^{-2}+xy^{-2}}{(x^2y)^{-2}}$$

$$= \left(\frac{\frac{1}{x^2}+\frac{x}{y^2}}{\frac{1}{x^4y^2}}\right)\left(\frac{x^4y^2}{x^4y^2}\right)$$

$$= \frac{x^2y^2+x^5}{1} = x^2(y^2+x^3)$$

(c) $$\frac{x-3}{x-\sqrt{3}}$$ (rationalize denominator)

$$= \left(\frac{x-3}{x-\sqrt{3}}\right)\left(\frac{x+\sqrt{3}}{x+\sqrt{3}}\right)$$

$$= \frac{x^2-3x+\sqrt{3}x-3\sqrt{3}}{x^2-3}$$