$\frac{a}{b} < \frac{c}{d}$ $\frac{a+b}{c+d}$

$ad < cb$

Math1090 Final Exam
Fall, 2009

Name Key

Instructions:

- Show all work, as partial credit will be given where appropriate.
- If no work is shown, there may be no credit given.
- All final answers should be written in the space provided on the exam and in simplified form.

DO *NOT* WRITE IN THIS TABLE!!!
(It is for grading purposes.)

Grade:

1	
2	
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Raw Total (out of 200 points)

Total (percentage)

1) (5 pts each part) If $f(x) = \frac{1}{x+2}$ and $g(x) = \sqrt{x-4}$, find each of the following, simplifying as far as possible:

(a) Domain of f(x) and g(x).

Domain of f(x): $x \neq 2$ or $(-\infty, 2) \cup (2, \infty)$

Domain of g(x): $x \geq 4$ or $[4, \infty)$

(b) $g(13)$

$$\sqrt{13-4} = \sqrt{9}$$

$g(13)$ = 3

(c) $\frac{f(x)}{g(x)}$

$$\frac{\frac{1}{x+2}}{\sqrt{x-4}} = \frac{1}{x+2} \cdot \frac{1}{\sqrt{x-4}}$$

$\frac{f(x)}{g(x)}$ = $\frac{1}{(x+2)\sqrt{x-4}}$

(Note: This is Problem 1 continued!)

$f(x)=\dfrac{1}{x+2}$ **and** $g(x)=\sqrt{x-4}$

(d) $(f \circ g)(x)$

$= f(g(x))$

$= \dfrac{1}{g(x)+2}$

$= \dfrac{1}{\sqrt{x-4}+2}$

$(f \circ g)(x) = \dfrac{1}{\sqrt{x-4}+2}$

(e) $f^{-1}(x)$

$y = \dfrac{1}{x+2}$

$\Rightarrow (x+2)\,y = 1$

$\Rightarrow xy + 2y = 1$

$\Rightarrow xy = 1 - 2y$

$\Rightarrow x = \dfrac{1-2y}{y}$

$f^{-1}(x) = \dfrac{1-2x}{x}$

2) (15 pts) (a) Find the slope of the line that goes through the points (-1, 4) and (5, 2).

$$\frac{4-2}{-1-5} = \frac{2}{-6} = -\frac{1}{3}$$

slope = $-1/3$

(b) Find the equation of the line that goes through the point (7, -5) and is perpendicular to the line from part (a) above. *(Give answer in slope-intercept form.)*

$\perp$ slope: 3

$y = 3x + b$

$\Rightarrow -5 = 3(7) + b$

$\Rightarrow -5 = 21 + b$

$\Rightarrow -26 = b$

line: $y = 3x - 26$

3) (10 pts) Solve the following equation. (Give exact answers.)

$$\frac{2x}{x-1}=5-\frac{1}{3x-3}$$

$$\Rightarrow (3x-3)\cdot\frac{2x}{x-1} = (3x-3)\left(5-\frac{1}{3x-3}\right)$$

$$\Rightarrow 3\cdot(x-1)\cdot\frac{2x}{x-1} = 5\cdot(3x-3) - 1$$

$$\Rightarrow 3\cdot 2x = 15x - 15 - 1$$

$$\Rightarrow 6x = 15x - 16$$

$$\Rightarrow 16 = 9x$$

$$\Rightarrow x = 16/9 \quad (\text{in domain of orig. expr.})$$

x = $16/9$

4) (15 pts) The startup costs for a street vendor are \$2000, and his cost per unit to produce food is \$1. Demand for his product dictates that the price at which q thousand units would be sold is $4-q$. Therefore, if q is the number of thousands of units sold, and cost and revenue are measured in thousands of dollars, the cost function is $C(q)=2+q$ and the revenue function is $R(q)=(4-q)q$.

(a) At which price(s) will the vendor break even? What quantities (in thousands) will he sell at these prices?

$$P(q) = R(q) - C(q)$$
$$= (4-q)q - (2+q) = 4q - q^2 - 2 - q$$
$$= -q^2 + 3q - 2$$
$$0 = -q^2 + 3q - 2 \Rightarrow q^2 - 3q + 2 = 0 \Rightarrow (q-1)(q-2) = 0$$
$$((p = 4 - q))$$

Break even price: \$3 or \$2 Break even quantity: 1 or 2 (thousand)

(b) What is the maximum profit? How many thousand units are sold to attain this profit? What is the price?

max: halfway between break-even: $1\frac{1}{2}$ th. units

price: $4 - 1\frac{1}{2}$

prof: $P(1\frac{1}{2}) = -(1\frac{1}{2})^2 + 3(1\frac{1}{2}) - 2$
$$= -2\tfrac{1}{4} + 4\tfrac{1}{2} - 2$$
$$= \tfrac{1}{4}$$

Maximum profit: \$0.25

Number of units sold for maximum profit: $1\frac{1}{2}$

price for maximum profit: \$2.50

5) (10 pts) Solve this system of equations using any method, and ALL work to support your answer must be shown here.

$$2x-3y+2z=5$$
$$x-2y+z=1$$
$$2y-z=4$$

$$\left[\begin{array}{ccc|c} 2 & -3 & 2 & 5 \\ 1 & -2 & 1 & 1 \\ 0 & 2 & -1 & 4 \end{array}\right] \hookrightarrow \begin{array}{c} \mathrm{II} \\ \mathrm{I} \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 2 & -3 & 2 & 5 \\ 0 & 2 & -1 & 4 \end{array}\right]$$

$(2\mathrm{I}: \ 2 \quad -4 \quad +2 \mid 2$

$$\hookrightarrow \begin{array}{c} \\ \mathrm{II}-2\mathrm{I} \\ \\ \end{array} \left[\begin{array}{ccc|c} 1 & -2 & 1 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 2 & -1 & 4 \end{array}\right] \Rightarrow \boxed{y=3}$$

$$\Rightarrow 2(3)-z=4$$

$$\Rightarrow \boxed{z=2}$$

$$\Rightarrow x-2(3)+2=1$$

$$\Rightarrow \boxed{x=5}$$

solution: (5, 3, 2) or x=5, y=3, z=2.

6) (5 pts each) Given the matrices A, B, and C, compute the following, if possible. If it's not possible, state the reason why.

$$A=\begin{bmatrix}2 & 1 & 3\\ 0 & 5 & -1\end{bmatrix}, \quad B=\begin{bmatrix}4 & 0\\ -3 & -2\end{bmatrix}, \quad C=\begin{bmatrix}7 & 0\\ 2 & 6\\ -1 & 4\end{bmatrix}.$$

(a) AB

$$\begin{bmatrix}2 & 1 & 3\end{bmatrix}\begin{bmatrix}4\\ -3\end{bmatrix}$$

X

AB = not possible, sizes not compatible

(b) B^{-1}

$$\frac{1}{4\cdot(-2)-(-3)\cdot 0}\begin{bmatrix}-2 & 0\\ 3 & 4\end{bmatrix}$$

B^{-1} = $-\frac{1}{8}\begin{bmatrix}-2 & 0\\ 3 & 4\end{bmatrix}$

(Note: This is Problem 6 continued.)

$$A=\begin{bmatrix}2 & 1 & 3\\ 0 & 5 & -1\end{bmatrix},\quad B=\begin{bmatrix}4 & 0\\ -3 & -2\end{bmatrix},\quad C=\begin{bmatrix}7 & 0\\ 2 & 6\\ -1 & 4\end{bmatrix}.$$

(c) BA

$$\begin{bmatrix}4 & 0\\ -3 & -2\end{bmatrix}\begin{bmatrix}2 & 1 & 3\\ 0 & 5 & -1\end{bmatrix} = \begin{bmatrix}8 & 4 & 12\\ -6 & -13 & -7\end{bmatrix}$$

$BA =$ $\begin{bmatrix}8 & 4 & 12\\ -6 & -13 & -7\end{bmatrix}$

(d) $3A - C^T$

$$= \begin{bmatrix}6 & 3 & 9\\ 0 & 15 & -3\end{bmatrix} - \begin{bmatrix}7 & 2 & -1\\ 0 & 6 & 4\end{bmatrix}$$

$3A - C^T =$ $\begin{bmatrix}-1 & 1 & 10\\ 0 & 9 & -7\end{bmatrix}$

7) (15 pts) Find the maximum value of the function $f = 2x - 4y + 15$ subject to the following constraints.

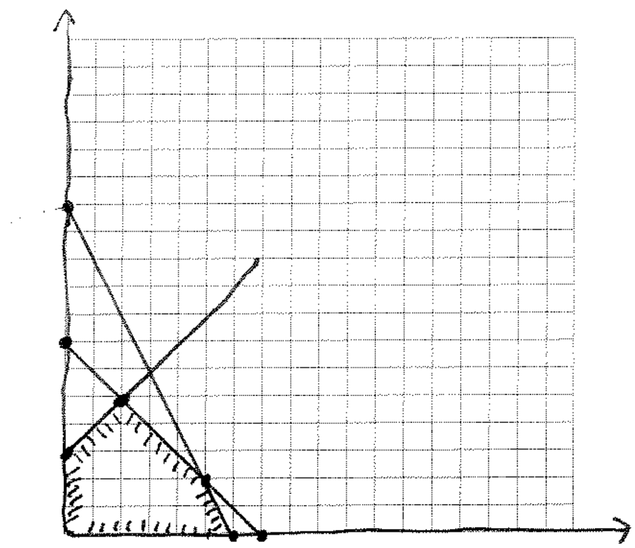
$y - x \leq 3$ ⇒ $y \leq x + 3$

$x + y \leq 7$ – int. $(0,7)$ & $(7,0)$

$2x + y \leq 12$ – int. $(0,12)$ & $(6,0)$

$x \geq 0$, $y \geq 0$

(a) Shade in the solution region.



(b) Find the vertices (or corners) of the solution region.

Vertices (or corners): $(0,3)$, $(6,0)$, $(2,5)$, $(5,2)$

(c) What is the maximum value of f? from form of f, want x big w/ y small

Max value: 27

(d) At what vertex (corner) does the maximum value of f occur?

vertex (corner) for max f value: $(6,0)$

8) (10 pts each part) Solve for x. (Show all work without a calculator.)

(a) $\ln x + \ln(x+2) = \ln 35$

$\Rightarrow \ln x(x+2) = \ln 35$

$\Rightarrow x(x+2) = 35$

$\Rightarrow x = 5$ or -7

but $\ln(-7) + \ln(-7+2) = \ln 35$

not defined.

$x =$ 5

(b) $5(2^{3x}) - 15 = 305$

$\Rightarrow 2^{3x} - 3 = 61$

$\Rightarrow 2^{3x} = 64 = 2^6$

$\Rightarrow 3x = 6$

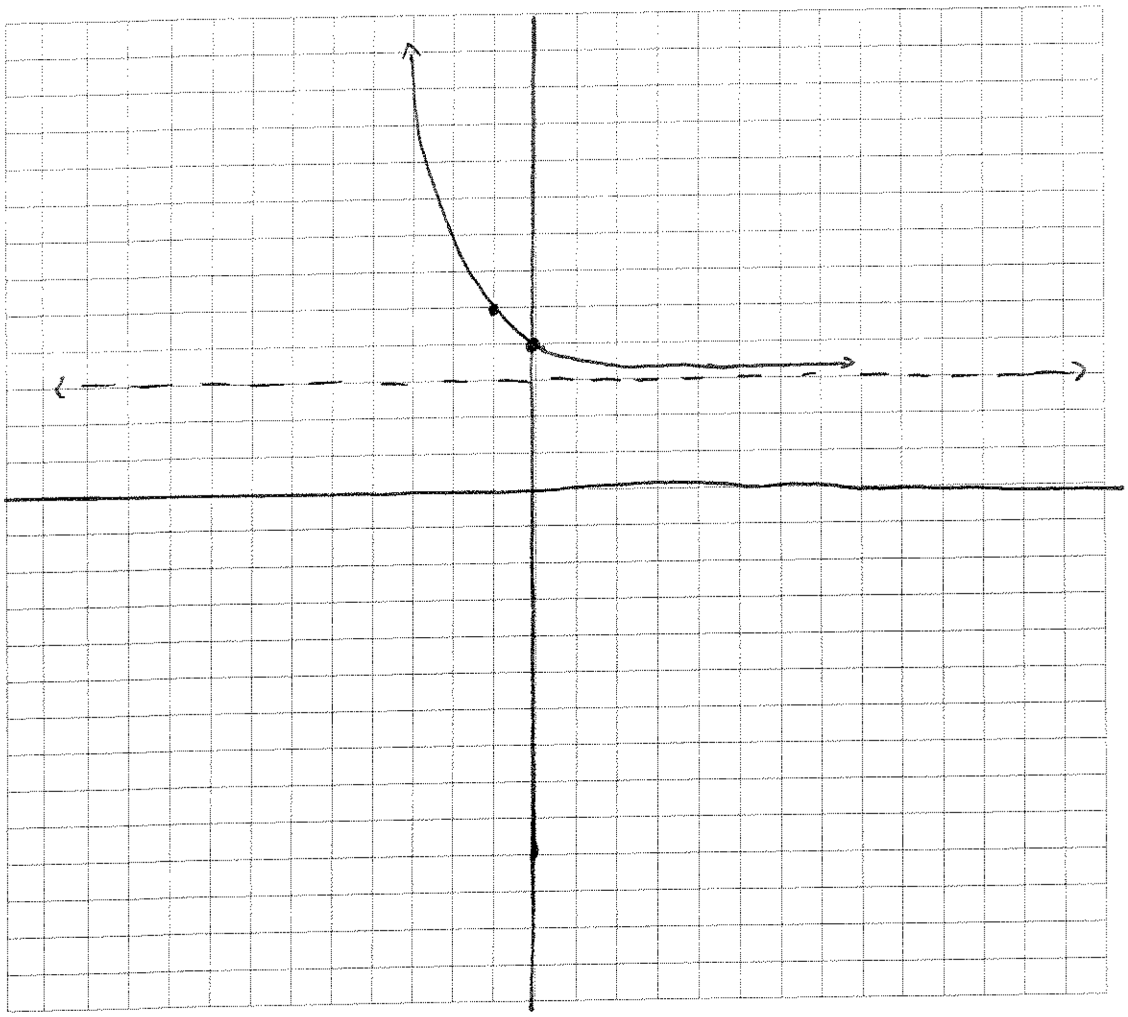
$\Rightarrow x = 2$

$x =$ 2

9) (10 pts each part) Sketch the graph of the given functions in the xy-plane. **Clearly label (1) two points on each graph and (2) line equations of any asymptotes.**

(a) $y=\left(\frac{1}{2}\right)^x+3$

$\left(\frac{1}{2}\right)^0+3=1+3=4$; $\left(\frac{1}{2}\right)^{-1}+3=2+3=5$



Two points on graph: $(0,4)$

$(-1,5)$

$\left(\frac{1}{2}\right)^x>0$

$\Rightarrow \left(\frac{1}{2}\right)^x+3>3$

Vertical Asymptote (if any): none

Horizontal Asymptote (if any): $y=3$

(Problem 9 instructions: Sketch the graph of the given function in the xy-plane. **Clearly label (1) two points on each graph and (2) line equations of any asymptotes.**)

(b) $y = \log_5(x) - 2$

$\log_5(1) - 2 = 0 - 2 = -2$

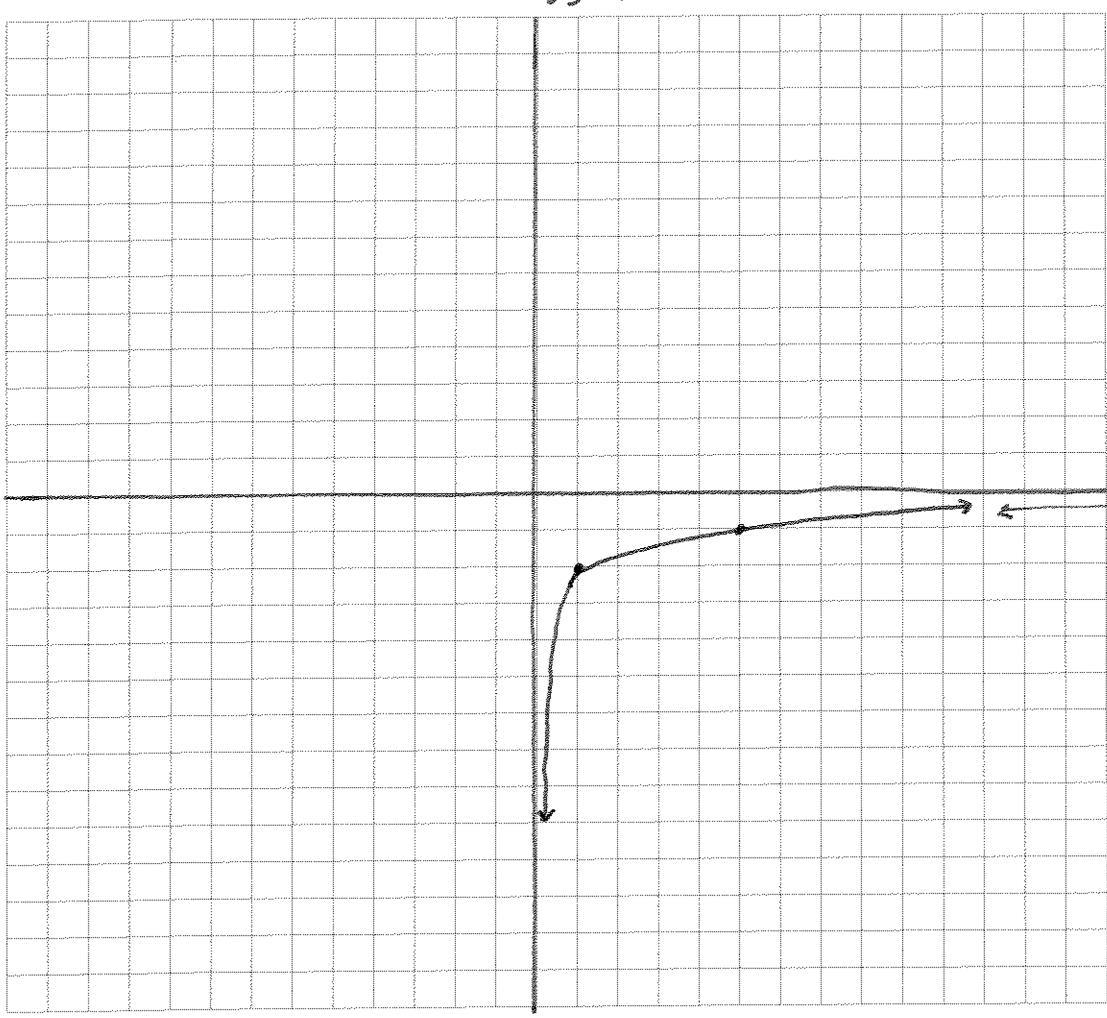
$\log_5(5) - 2 = 1 - 2 = -1$

$\log_5 x - 2 = 0$

$\Rightarrow \log_5 x = 2$

$\Rightarrow \underline{x = 25}$

domain of $\log_5 x$: $x > 0$



not asymptote to x-axis

Two points on graph: $(1, -2)$

$(5, -1)$

Vertical Asymptote (if any): $x = 0$ (y-axis)

Horizontal Asymptote (if any): none

10) (15 pts) Given $y=-x^2+2x+3$, as the equation of a parabola, answer the following questions.

(a) Find the coordinates of the vertex.

$$x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1; \quad y = -(1)^2+2(1)+3 = 4$$

Vertex: $(1,4)$

(b) Find the x-intercept(s), if any.

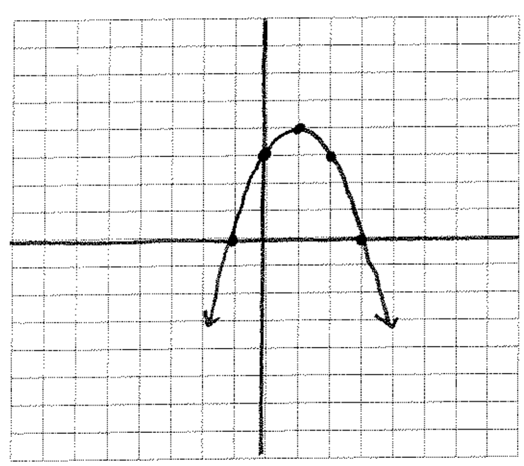
$$0 = -x^2+2x+3 \Rightarrow x^2-2x-3=0 \Rightarrow (x-3)(x+1)=0$$

x-intercept(s): $(3,0)$, $(-1,0)$

(c) Find the y-intercept.

y-intercept: $(0,3)$

(d) Sketch the graph.



11) (15 pts) Martha and Paul buy a house for \$175,000. They have a \$25,000 down payment and expect to amortize the rest of the debt with monthly payments over the next 30 years. The interest on the debt is 5.1% compounded monthly.

(a) What are the monthly payments?

debt = 150,000 ; mth. rate $= \frac{0.051}{12} = 0.00425$

$$150\,000 = R\left[\frac{1-(1.00425)^{-360}}{0.00425}\right]$$

$$\Rightarrow 637.5 = R\left(1-(1.00425)^{-360}\right)$$

$$\approx R\cdot(0.78)$$

Monthly payments: \$ 814.42

(b) Find the total amount of house loan payments (excluding down payment).

360 pmts. @ \$ 814.42

Total paid: \$293,192.88

(c) Find the total amount of interest paid on this loan.

total paid − principal

293,192.88 − 150,000

Total Interest paid: \$ 143,192.88

12)(10 pts each part) Kilam is 20 years old. Investing in mutual funds, he can earn 8% interest, compounded quarterly.

(a) In order to have $50,000 in the account to buy a sports car when he is 30 years old, how much money must he deposit at the end of each quarter?

fut. value. quarterly rate: $\frac{0.08}{4} = 0.02$

$$50{,}000 = R\left[\frac{(1.02)^{40} - 1}{0.02}\right]$$

$$\Rightarrow \quad 1000 \approx R(1.21)$$

Quarterly deposits: $ 827.79

(b) When Kilam turns 30, he decides not to buy the car, but leaves his money in the account (still earning compounded interest), now *withdrawing* $2,000 at the end of each quarter. After how many quarters does he run out of money?

pres. value.

$$50000 = R\left[\frac{1 - (1.02)^{-n}}{0.02}\right]$$

$$\Rightarrow \quad 1000 = 2000\left(1 - (1.02)^{-n}\right)$$

$$\Rightarrow \quad \frac{1}{2} = 1 - (1.02)^{-n}$$

$$= \quad (1.02)^{-n} = \frac{1}{2} \quad \Rightarrow \quad n \ln 1.02 = \ln 2$$

$$\Rightarrow \quad n = \frac{\ln 2}{\ln 1.02}$$

Number of quarters it takes for him to run out of money from fund: ≈ 35

(technically 36)