1.1 Linear Equations and Inequalities

Vocab

equation	vs. expression
vs. identity	equivalent equations (same as "keeping scale balanced")

★ Solve equations and ★ simplify expressions

Ex 1 Solve

(a) $3x+22=7x+2$

(b) $\frac{2x}{3}-1=\frac{x-2}{2}$

1.1 (cont)

Ex 2 Solve these rational eqns (that turn in to linear eqns). Note: check domain.

(a) $\frac{2x}{x-3} = 4 + \frac{6}{x-3}$

(b) $\frac{3}{x} + \frac{1}{4} = \frac{2}{3} + \frac{1}{x}$

vocab
Rational eqn =>
domain =>

1.1 (cont)

Ex 3 Solve for y in terms of x.

$$\frac{3x}{2} + 5y = \frac{1}{3}$$

linear Inequalities ⇒ use same strategies used in solving linear equations ... except if multiply or divide by negative #, switch sign

Ex 4 $2(x+1) > x-1$

Ex 5 $\frac{-2x}{5} \le -10 - x$

1.1 (cont)

Ex 6 Suppose a professor counts the final exam as being equal to each of the other tests in her course, and she will also change the lowest test score to match the final exam score if the final exam score is higher. If a student's four test scores are 83, 67, 52 and 90, what is the lowest score the student can earn on the final exam & still obtain at least an 80 average for the course?

1.2 Functions

Relation vs. **Function**

Relation	Function
defined by set of ordered pairs (x, y)	a relation such that every input has exactly one output

domain $\Rightarrow$ set of inputs **range** $\Rightarrow$ set of outputs

Ex 1 Relation or Function?

(a) $x =$ person $y =$ car owned by that person

(b) $x =$ person $y =$ their kid

(c) $x =$ person $y =$ their mom

(d) $x =$ student $y =$ grade in their Math 1090 class

Vertical line test

If we graph all the ordered pairs (of a relation) on a Cartesian coordinate system, and every vertical line goes through the graph at most one time, then it's a function.

1.2 (cont)

Ex 2 Are these functions? Identity domains.

(a) $y = f(x) = 6x^2$

(b) $y^2 = 4x^2$

Ex 3 Evaluate, given $f(x) = 3x^2 - 2x$

(a) $f(-3)$

(b) $f(2)$

(c) For $f(x)$, what is the domain?

1.2 (cont)

Operations w/ Functions

sum $(f+g)(x) = f(x) + g(x)$

difference $(f-g)(x) = f(x) - g(x)$

quotient $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$

product $(f \cdot g)(x) = f(x)g(x)$

Composite Functions

Let $f(x)$ & $g(x)$ be functions. Then,

$$(f \circ g)(x) = f(g(x))$$

and $(g \circ f)(x) = g(f(x))$

Note

Don't confuse $\bullet$ with $\circ$

Ex 4 For $f(x) = \frac{1}{x^3}$ and $g(x) = 4x+1$,

find (a) $f(g(x))$

(b) $g(f(x))$

1.2 (cont)

Ex 5 Find domain and range for

(a) $f(x) = y = \sqrt{x+1}$

(b) $y = x^2 + 4$

Ex 6 For $f(x) = 3x^2 - 6x$

(a) find $f(x+h)$

(b) $f(x) + h$

1.3 Linear Functions

Defn
linear function
any function of form $y = f(x) = ax + b, \ a, b \in \mathbb{R}$

Vocab intercept : y-intercept
x-intercept

slope : "steepness" $\Rightarrow$ $m = \dfrac{y_1 - y_2}{x_1 - x_2} = \dfrac{y_2 - y_1}{x_2 - x_1}$

(x_1, y_1) and (x_2, y_2) pts on line

(note: slope of a horizontal line =
slope of a vertical line =)

(⊥) Perpendicular lines

lines have slopes that are negative reciprocals of each other

II) Parallel lines

lines have same slope

ex $y = 3x + 2$
$y = 3x - 1$
parallel

ex $y = 4x - 1$
$y = \frac{-1}{4}x + 2$
perpendicular

1.3 (cont)

Two main eqns of lines:

Point-slope

Given pt (x_1, y_1) and slope m.

$$y - y_1 = m(x - x_1)$$

slope-intercept

given slope m and y-intercept $(0, b)$

$$y = mx + b$$

Ex 1 Find slope of line through $(-4, 2)$ and $(1, 5)$.

Note: synonyms for slope $\Rightarrow$
"rate of change"
"speed"
"velocity"

Ex 2 Find slope and y-intercept for $3x + 2y = 5$.

1.3 (cont)

Ex 3 Write the eqn of the line (in slope-intercept form).

(a) $m=4$, y-intercept $(0,-1)$

(b) $m=-1$, thru pt $(-3,2)$

(c) thru pts $(-1,4)$ and $(2,1)$

1.3 (cont)

Ex 4 Write eqn of line thru $(6, -4)$ and perpendicular to line $4x-5y=6$.

(#50)
Ex 5 The percent p of high school seniors who smoke cigarettes can be described by $p = 32.88 - 0.03t$ where t = # yrs past 1975.

(a) Find slope + p-intercept.

(b) Interpret meaning of slope (as rate of change).

(c) Interpret meaning of p-intercept.

1.5 Solutions of Systems of Linear Equations

<u>vocab</u> system of equations ⇒

solutions ⇒

Methods for solving

1. Graphing (not very reliable)
2. Substitution (always works)
3. Elimination (useful for linear eqns)

→ "Equivalent systems"

(a) interchange two eqns

(b) multiply by nonzero constant

(c) a multiple of one eqn is added to another eqn.

Possible solutions (for system of 2 eqns w/ 2 variables)

1. one solution 
2. no solution 
3. infinitely many solutions 

1.5 (cont)

Ex 1 Solve $2x - y = 3$

$4x - y = 5$

Substitution | Elimination

Ex 2 Solve $x + 2y = 3$

$3x + 6y = 6$

1.5 (cont)

Ex 3 Solve
$$2x - y = 5$$
$$-6x + 3y = -15$$

Ex 4 Solve
$$x + 3y - 8z = 20$$
$$y - 3z = 11$$
$$2y + 7z = -4$$

1.5 (cont)

Ex 5 (#36) A woman has \$500,000 invested in two rental properties. One yields an annual return of 10% of her investment, + the other returns 12% per year on her investment. ~~Her total~~ annual return from the two properties is \$53,000. Let x = amt invested at 10% + y = amt invested at 12%. How much is invested in each property?

1.6 Applications of Functions (in Business and Economics)

Cost, Revenue, Profit

x = # units produced & sold

$C(x)$ = cost $\quad$ $R(x)$ = revenue $\quad$ $P(x)$ = profit

$$P(x) = R(x) - C(x)$$

usually $\quad C(x)$ = variable costs + fixed costs

Ex 1 Suppose a computer manufacturer has a total cost of $C(x) = 85x + 3300$ and revenue of $R(x) = 385x$.

(a) What is the profit fn?

(b) What is profit on 351 items?

(c) How many items must be sold to break even?

1.6 (cont)

Ex1 (cont)

(d) What is the slope of the profit fn?

(e) What is the marginal revenue?

(f) marginal profit?

Note

"marginal" = how much something changes for one more unit sold.

marginal cost = $\overline{MC}$

marginal profit = $\overline{MP}$

marginal revenue = $\overline{MR}$

★ basically it's the slope!

1.6 (cont)

Ex 2 A manufacturer sells watches for \$50 per unit. The fixed costs related to this product are \$10,000 per month and the variable costs are \$30 per unit.

(a) write the revenue and cost fns.
(Let $x=$ # units produced/sold per month)

(b) How many watches need to be sold to break even?

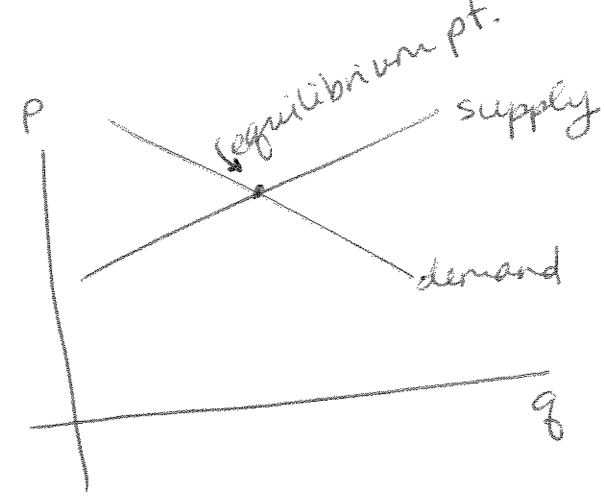
(c) what is the profit fn?

1.6 (cont)

Supply and Demand

p = price

q = quantity



Ex 3 If the supply fn for a phone is $35p - 20q = 350$ and the demand fn is $p + 2q = 100$, compare the quantity demanded vs. supplied, (a) when the price is \$14. Are there surplus phones or not enough to meet demand? (b) what is the equilibrium pt?

1.6 (cont)

Ex 4 Retailers will buy 45 cordless phones from a wholesaler if the price is \$10 each but only 20 if the price is \$60. The wholesaler will supply 35 phones at \$30 each and 70 at \$50 each. Assuming supply and demand are linear, find the market equilibrium pt.