2.1 Quadratic Equations

Quadratic Eqn ⇒ eqn of the form $ax^2+bx+c=0$ $a,b,c \in \mathbb{R}$, $a \neq 0$

Zero Product Property

For $a,b \in \mathbb{R}$, $ab=0 \iff a=0$ or $b=0$, or $a=b=0$.

4 ways to solve quadratic eqns

only work sometimes:

1. square root technique (only works if $b=0$)
2. factoring

always work:

3. completing the square
4. quadratic formula $x = \dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

Derive quadratic formula ⇒

$$ax^2+bx+c=0$$

2.1 (cont)

Ex 1 Solve

(a) $25x^2 - 49 = 0$

(b) $x^2 - 4x = 3x^2$

2.1 (cont)

Ex 2 Solve.

(a) $x^2 + 17x = 8x - 14$

(b) $x^2 + 2x + 4 = 0$

(c) $(x+1)^2 = 2$

2.1 (cont)

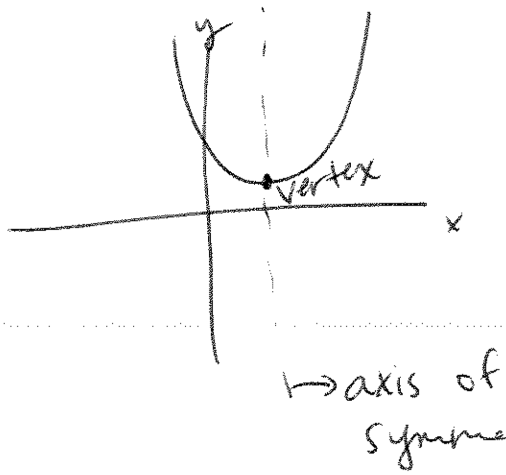
Ex 3 Solve.

(a) $(y-2)^2 - 5(y-2) - 24 = 0$

(b) $\dfrac{5}{t+4} - \dfrac{3}{t-2} = 4$

2.2 Quadratic Functions: Parabolas

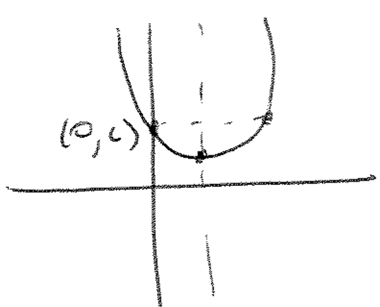
Quadratic Fn $\Rightarrow$ $y = f(x) = ax^2 + bx + c$
(a quadratic eqn in two variables)
when we graph all the solutions to this,
the points form a parabola.



For $y = ax^2 + bx + c$,
if $a > 0$, ∪ concave up
if $a < 0$, ∩ concave down

Let's figure out where the vertex is (algebraically) so we can always find it.

if we plug in $x = 0$, we get pt on y-axis $\Rightarrow$
$x = 0 \Rightarrow y = a(0^2) + b(0) + c \iff y = c$, i.e. parabola goes thru $(0, c)$



We can see, by symmetry of parabola, that there is another pt whose y-value is c.

2.2 (cont)

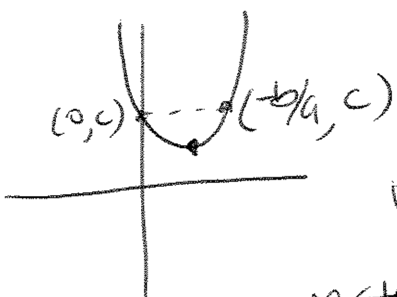
$c = ax^2 + bx + c \iff 0 = ax^2 + bx$

$0 = x(ax+b)$

$x = 0$, or $ax + b = 0$

$ax = -b$

$x = -b/a$



You can see (from symmetry) that x-value of vertex is halfway between 0 and $-b/a$, i.e. $x = -b/2a$.

$\Rightarrow$ **vertex at $\left(-b/2a,\ f(-b/2a)\right)$**

axis of symmetry $x = -b/2a$

Ex 1 For $y = -2x^2 - 4x + 6$

(a) find vertex

(b) Is the vertex a min or max pt?

2.2 (cont)

Ex 2 For $y = x^2 - 6x + 9$,

(a) find vertex.

(b) Is it a min or max pt?

(c) find zeroes of graph.

(d) sketch the graph.

2.2 (cont)

Ex 3 Describe shifting for $y = (x-10)^2 + 1$.

Ex 4 Find the average rate of change of $y = \frac{1}{2}x^2 + 3x + 8$ between $x = 2$ and $x = 4$.

Ex 5 If 100 ft of fencing is used to enclose a rectangular yard, then the resulting area is given by $A = x(50-x)$. Graph this equation, and give the length and width that maximize area.

2.3 Business Applications of Quadratic Fns

Supply Demand + Market Equilibrium

Ex 1 If the supply function for a commodity is $p = q^2 + 8q + 20$ and the demand function is $p = 100 - 4q - q^2$, find the equilibrium quantity and equilibrium price. (Sketch both curves.)

2.3 (cont)

Ex 2 For the last example, if a $8 tax is placed on production & passed through the supplier, find the new equilibrium pt.

2.3 (cont)

Break-Even Pts and Maximization

Ex 3 If a company has total costs $C(x) = 1600 + 1500x$ and total revenue is $R(x) = (1600 - x)x$, find the break even pts.

> Break Even pts occur when
> $R(x) = C(x)$
> $\iff P(x) = 0$

2.3 (cont)

Ex 4 Find maximum revenue given
$R(x) = 1600x - x^2$.

Ex 5 Suppose a company has fixed costs of \$300 and variable costs of $\frac{3}{4}x + 1460$ dollars per unit, where x = total # units produced. Suppose further that its selling price is $1500 - \frac{1}{4}x$ dollars per unit.

(a) Find break even pts.

2.3 (cont)

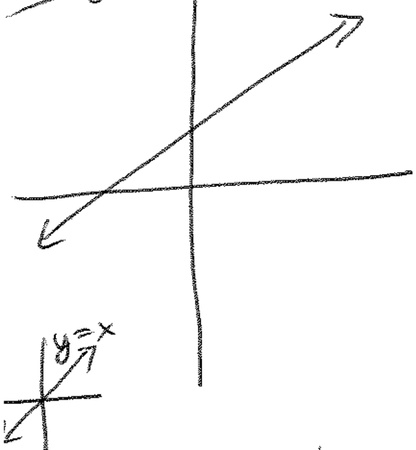
Ex 5 (cont)

(b) Find max revenue

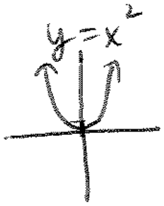
(c) Find max profit, and price that yields it.

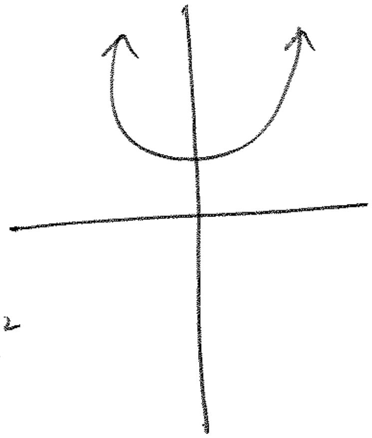
2.4 Special Fns and Their Graphs

Polynomials

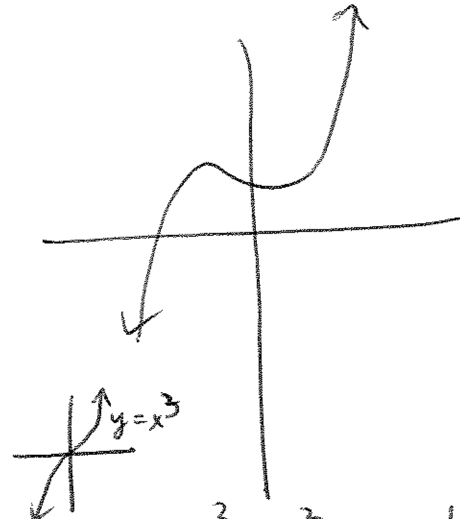


$y = mx + b$



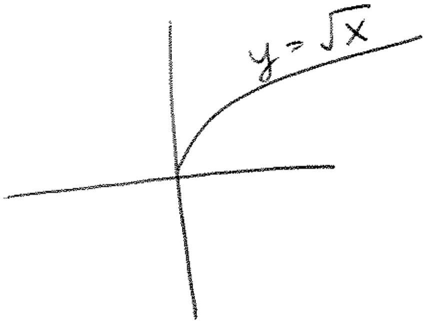


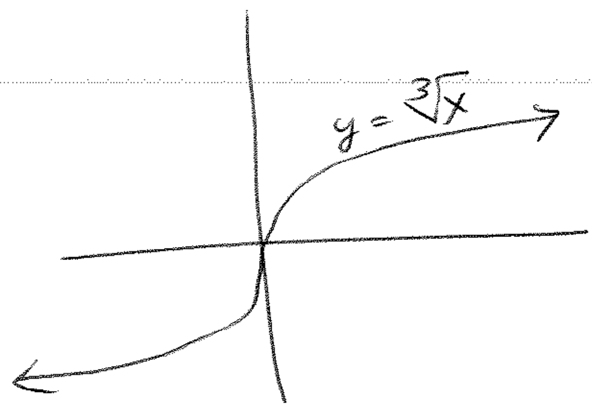
$y = ax^2 + bx + c$



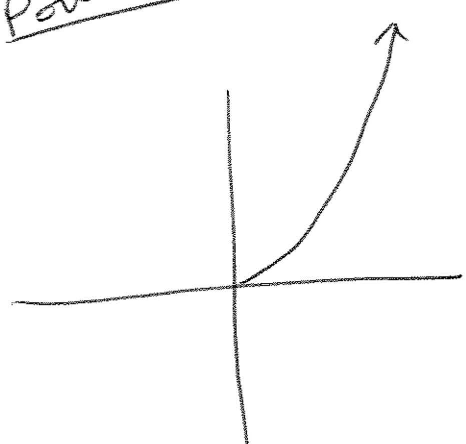
$y = ax^3 + bx^2 + cx + d$

Radical

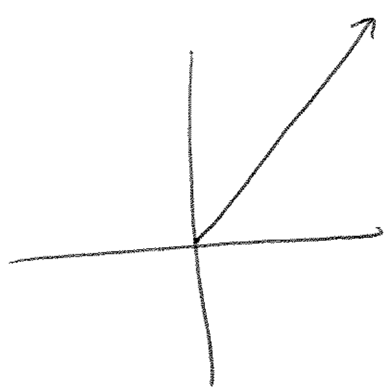




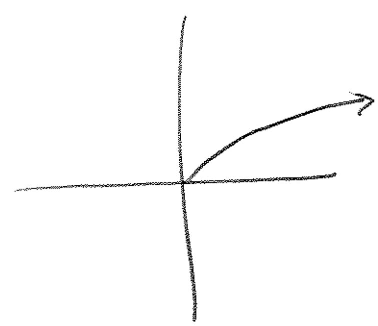
Powers



$y = x^b$
$b > 1$



$y = x^1$



$y = x^b$
$0 < b < 1$

2.4 (cont)

Shifting Graphs

$y = f(x+h) + k$ shifts up by k units
(assume $h, k > 0$) and left by h units

Ex 1 Describe shifting of $f(x) = (x-2)^2 + 3$ compared to base graph of $y = x^2$.

Piecewise Functions

ex $f(x) = |x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$

Ex 2 Graph $f(x) = \begin{cases} x^2+5 & x \geq 2 \\ x+3 & 0 \leq x < 2 \\ -x^2-1 & x < 0 \end{cases}$

2.4 (cont)

Rational Functions $\Rightarrow$ $\frac{f(x)}{g(x)}$ where $f(x)$ and $g(x)$ are both polynomials.

Asymptotes

① vertical $\Rightarrow$ "restriction" of what x-values graph cannot have (comes from domain)

② horizontal $\Rightarrow$ "description" of what happens to graph as $x \to \pm\infty$

Ex 3 Graph $f(x) = \frac{x-3}{x+2}$

2.4 (cont)

Ex 4 Given function $y = \begin{cases} \frac{1}{2}x + 4 & x < 0 \\ 4 - x & 0 \le x < 4 \\ 0 & x > 4 \end{cases}$

(a) find $y(1)$

(b) find $y(3.9)$

(c) find $y(-4)$

Ex 5 Graph $y = (x+2)^3 - 3$

2.6 Composite and Inverse Functions

Composite Functions

Given $f(x)$ and $g(x)$, $(f \circ g)(x) = f(g(x))$.

Ex1 Given $f(x) = 2x+8$, $g(x) = \frac{1}{x^3}$,

(a) find $(f \circ g)(x)$.

(b) find $(g \circ f)(x)$.

Ex 2 Find fns $f(x)$ and $g(x)$ $\ni$ $(f \circ g)(x) = \frac{1}{5x^3+4}$.

2.6 (cont)

Inverse Functions

an inverse function basically "undoes" what original function did to input

notation ⇒ $f^{-1}(x)$ (read "f inverse of x")

$\Rightarrow f(f^{-1}(x)) = x = f^{-1}(f(x))$

Ex 3 Are $f(x) = 5x-1$ and $g(x) = \frac{x+1}{5}$ inverse functions?

Finding an inverse function ⇒ basically you need to undo the operations in the opposite order to how they were done

2.6 (cont)

Ex 4 Find the inverse function for

$$y = \frac{(x-2)^3}{4} + 5$$

Does every function have an inverse?

A function has an inverse if it passes the horizontal line test! (i.e. if it is one-to-one ⇒ every input has exactly one output + every output has also only one input)

2.6 (cont)

inverse fns are mirror images across line $y = x$.

Ex 5 Does $y = x^2$ have inverse function?

Ex 6 Is function defined by $\{(1,3), (6,2), (4,3)\}$ one-to-one?

Ex 7 To convert from celsius to Fahrenheit, you can use $F = \frac{9}{5}C + 32$. Change this formulla to allow you to convert from °F to °C.