Problem 1 (Fractions). Simplify:

$$\frac{\frac{2}{3}-\frac{1}{4}}{\frac{3}{4}+\frac{1}{3}}$$

$$\frac{\frac{2}{3}-\frac{1}{4}}{\frac{3}{4}+\frac{1}{3}} = \frac{\frac{2\cdot 4-1\cdot 3}{12}}{\frac{3\cdot 3+1\cdot 4}{12}} = \frac{\frac{8-3}{12}}{\frac{9+4}{12}} = \frac{\frac{5}{12}}{\frac{13}{12}} = \frac{5\cdot 12}{12\cdot 13} = \frac{5}{13}$$

Simplified fraction: $\frac{5}{13}$

Problem 2 (A Linear Equation). Solve the equation:

$$5x - 1 = 8 - 3(x - 2)$$

$5x-1 = 8-3x+6$

$5x-1 = 14-3x \quad /+3x$

$8x-1 = 14 \quad /+1$

$8x = 15 \quad /\div 8$

$x = \frac{15}{8}$

$x = \frac{15}{8}$

Problem 3 (Quadratic Equation). Find all solutions of the equation:

$$x^2 - 3x - 6 = 0$$

(1) QUADRATIC FORMULA

$$x_{1/2} = \frac{3 \pm \sqrt{(-3)^2 - 4 \cdot 1 \cdot (-6)}}{2 \cdot 1} = \frac{3 \pm \sqrt{9+24}}{2 \cdot 1} = \frac{3 \pm \sqrt{33}}{2}$$

(2) COMPLETE SQUARE

$$x^2 - 3x = 6 \quad / + \left(-\frac{3}{2}\right)^2 = \frac{9}{4}$$

$$x^2 - 3x + \frac{9}{4} = 6 + \frac{9}{4}$$

$$\left(x - \frac{3}{2}\right)^2 = \frac{33}{4} \quad / \sqrt{\ }$$

$$x - \frac{3}{2} = \pm\frac{\sqrt{33}}{2}$$

$$x = \frac{3}{2} \pm \frac{\sqrt{33}}{2}$$

$$x = \frac{3 \pm \sqrt{33}}{2}$$

Problem 4 (A rational equation). Find all solutions of:

$$\frac{4}{x-5} + \frac{1}{x-2} + 1 = 0 \quad / \cdot (x-5)(x-2)$$

Domain $x \neq 5$, $x \neq 2$

$$4(x-2) + 1(x-5) + (x-2)(x-5) = 0$$

$$4x - 8 + x - 5 + x^2 - 5x - 2x + 10 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x - 3 = 0$ or $x + 1 = 0$

$x = 3$ or $x = -1$

check

$$\frac{4}{3-5} + \frac{1}{3-2} + 1 = \frac{4}{-2} + \frac{1}{1} + 1 = -2 + 2 = 0 \ \checkmark$$

$$\frac{4}{-1-5} + \frac{1}{-1-2} + 1 = \frac{4}{-6} + \frac{1}{-3} + 1 = -\frac{2}{3} - \frac{1}{3} + 1 = -1 + 1 = 0 \ \checkmark$$

$$x = 3 \text{ OR } -1$$

Problem 5 (Polynomials). Write the following polynomial expression in standard form. What is its degree and its leading coefficient?

$$(x^3+2)(x-5)+7x+9$$

$x^4-5x^3+2x-10+7x+9 =$

$= x^4-5x^3+9x-1$

polynomial x^4-5x^3+9x-1
degree 4
leading coefficient 1

Problem 6 (Inequality). Solve the following inequality

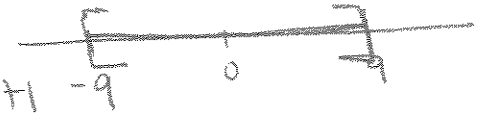
$|2x-1|-5 \le 4 \quad /+5$

$|2x-1| \le 9$

$-9 \le 2x-1 \le 9 \quad /+1$

$-8 \le 2x \le 10 \quad /\div 2$

$-4 \le x \le 5$



Answer $x \in [-4,5]$
or $-4 \le x \le 5$

For the next three questions let

$$f(x) = \frac{x^2+3}{x^2-4}$$

Problem 7 (Domain). What is the domain of f?

$x^2 - 4 \neq 0$

$(x-2)(x+2) \neq 0$

$x \neq \pm 2$

Domain $\mathbb{R} \setminus \{-2, 2\}$; OR all real #s except $-2, 2$.

Problem 8 (Evaluate). Find $f(-1)$.

$$f(-1) = \frac{(-1)^2+3}{(-1)^2-4} = \frac{1+3}{1-4} = \frac{4}{-3} = -\frac{4}{3}$$

$f(-1) = -\frac{4}{3}$

Problem 9 (Evaluate at an expression). Find $f(x+1)$ and express it in the standard form of a rational expression (quotient of two polynomials each given in standard form).

$$f(x+1) = \frac{(x+1)^2+3}{(x+1)^2-4} = \frac{x^2+2x+1+3}{x^2+2x+1-4} = \frac{x^2+2x+4}{x^2+2x-3}$$

$f(x+1) = \frac{x^2+2x+4}{x^2+2x-3}$

Problem 10 (Radical Equations). Solve the equation

$$\sqrt{x+2}+x+3=7.$$

$\sqrt{x+2}+x=4 \quad / -x$

$\sqrt{x+2}=4-x \quad / ^2$

$x+2=(4-x)^2$

$x+2=16-8x+x^2 \quad / -x-2$

$x^2-8x+16-x-2=0$

$x^2-9x+14=0$

$(x-7)(x-2)=0$

$x=7$ or $x=2$

check

(7) $\sqrt{7+2}+7+3=$

$=\sqrt{9}+7+3=$

$=3+7+3=13\neq 7$ ✗

(2) $\sqrt{2+2}+2+3=$

$=\sqrt{4}+5=2+5=7$

$x=$ 2

Problem 11 (Linear System). Solve the system

$$\begin{aligned} 4x &- y &&= 1 \\ 2x &+ y &&= 0 \end{aligned}$$

Show all your work, don't just give the answer.

$4x-y=1$

$2x+y=0$ } +

$6x=1 \quad / \div 6$

$x=\frac{1}{6}$

$2x+y=0$

$y=-2x$

$y=-2\cdot\frac{1}{6}=-\frac{1}{3}$

check: $2\cdot\frac{1}{6}-\left(-\frac{1}{3}\right)=\frac{2}{3}+\frac{1}{3}=1$ ✓

$x=\frac{1}{6}$

$y=\frac{-1}{3}$

Problem 12 (Straight Lines). Find an equation of the line that passes through (3,4) and has slope -2/3. Draw its graph.

$m = -\frac{2}{3}$

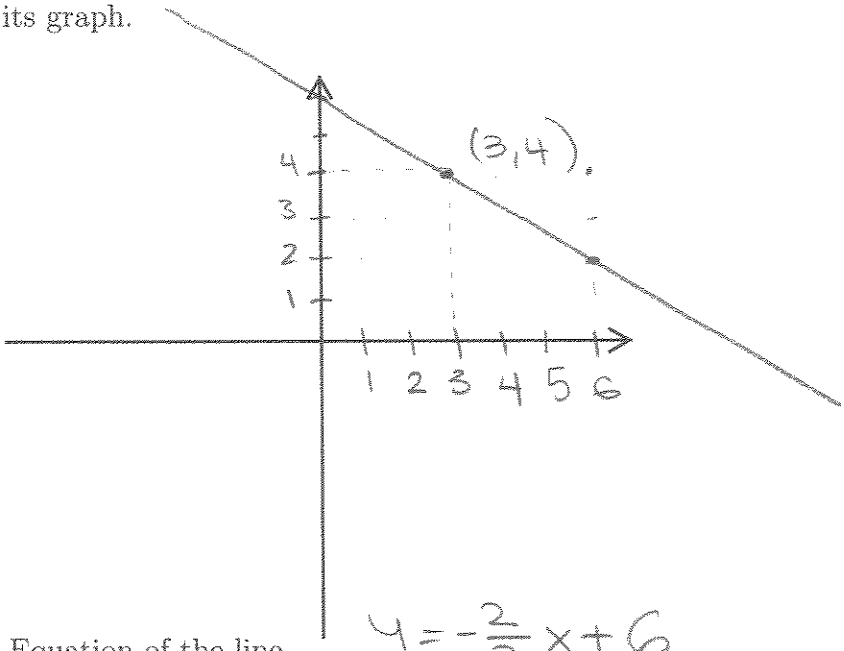
$\ell: \quad y = -\frac{2}{3}x + b$

(3,4) on ℓ:

$4 = -\frac{2}{3}\cdot 3 + b$

$4 = -2 + b \quad /+2$

$\boxed{b = 6}$



Equation of the line $y = -\frac{2}{3}x + 6$

Problem 13 (Distance). Find the distance between the points $(2, -1)$ and $(-3, 4)$.

$(2, -1)$

$(-3, 4)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} =$

$= \sqrt{(2-(-3))^2 + (-1-4)^2} =$

$= \sqrt{5^2 + (-5)^2} = \sqrt{25+25} = \sqrt{25 \cdot 2} = 5\sqrt{2}$

Distance is $5\sqrt{2}$

Problem 14 (Powers). Simplify (i.e., write with only positive exponents, such that x and w occur only once) the expression

$$\frac{(w^{\frac{3}{2}}x^{-4})^2}{(w^{-2}x^0x^{\frac{2}{3}})^{-3}}, \quad (x, w > 0)$$

$= \dfrac{\left(w^{\frac{3}{2}}\right)^2 \cdot \left(x^{-4}\right)^2}{\left(w^{-2} \cdot x^{\frac{2}{3}}\right)^{-3}} = \dfrac{w^{\frac{3}{2}\cdot 2} \; x^{-4\cdot 2}}{\left(w^{-2}\right)^{-3} \cdot \left(x^{\frac{2}{3}}\right)^{-3}} =$

$= \dfrac{w^3 \cdot x^{-8}}{w^6 \cdot x^{\frac{2}{3}\cdot(-3)}} = \dfrac{w^3 \; x^{-8}}{w^6 \; x^{-2}} = w^{3-6} \; x^{-8-(-2)} =$

$= w^{-3}x^{-6} = \dfrac{1}{w^3x^6}$

Answer $\dfrac{1}{w^3x^6}$

Problem 15 (Rational expression). Simplify the following expression (write as a single fraction)

$$\frac{3}{x-1} + \frac{2}{x+5} - \frac{1}{x+3}$$

$\dfrac{3}{x-1} + \dfrac{2}{x+5} - \dfrac{1}{x+3} = \dfrac{3(x+5)(x+3) + 2(x-1)(x+3) - (x-1)(x+5)}{(x-1)(x+5)(x+3)}$

$= \dfrac{3(x^2+8x+15) + 2(x^2+2x-3) - (x^2+4x-5)}{(x-1)(x+5)(x+3)} =$

$= \dfrac{3x^2+24x+45+2x^2+4x-6-x^2-4x+5}{(x-1)(x+5)(x+3)} =$

$= \dfrac{4x^2+24x+44}{(x-1)(x+5)(x+3)} =$

Answer $\dfrac{4x^2+24x+44}{(x-1)(x+5)(x+3)}$

Problem 16 (Word problem). A plumber is working on your toilet. He charges you \$40 for a house call that includes first half an hour of work. For every hour thereafter he charges \$25. How many hours was the plumber working on your toilet if the total bill was \$90?

t = # hours of work (after first half hour).

P = price

$P = 40 + 25t$

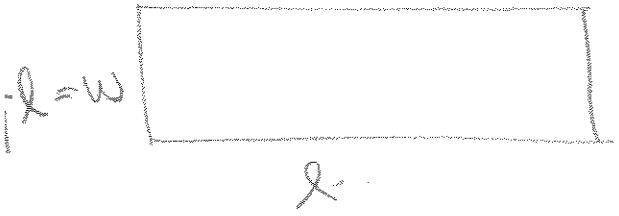
$90 = 40 + 25t \quad / -40$

$50 = 25t \quad / \div 25$

$\boxed{t = 2}$

Answer 2.5 hours

Problem 17 (Another Word Problem). The perimeter of a rectangle is 68 ft and its width is $\frac{8}{9}$ times its length. Find the dimensions of the rectangle.



$P = 68\text{ ft} = 2l + 2w$

$68 = 2l + 2 \cdot \frac{8}{9} l$

$68 = 2l + \frac{16}{9} l$

$68 = \frac{18+16}{9} l$

$68 = \frac{34}{9} l$

$l = \frac{68 \cdot 9}{34} = 18$

$w = \frac{8}{9} \cdot 18 = 16$

Length 18

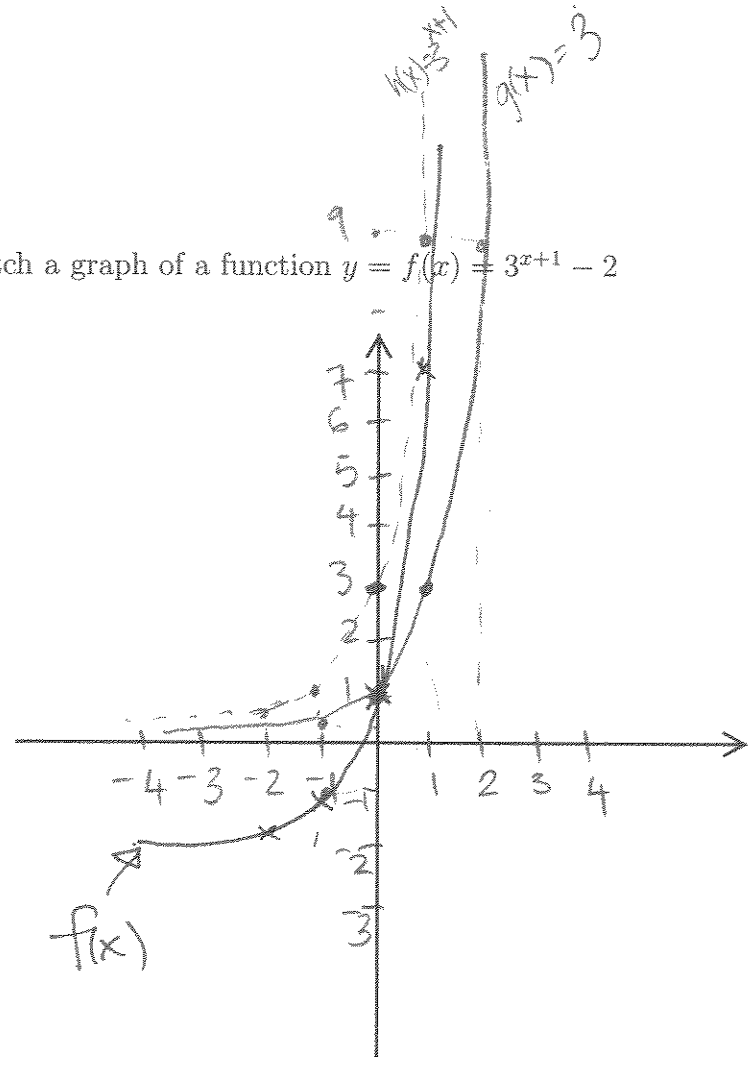
Width 16

check

$2 \cdot 18 + 2 \cdot 16 =$

$= 36 + 32 = 68$ ✓

Problem 18 (Graph). Sketch a graph of a function $y = f(x) = 3^{x+1} - 2$



Problem 19 (Inverse functions). Find the inverse function of $f(x) = 5x^3 + 4$ (you must show enough work: answer only will yield minimum credit.)

$y = 5x^3 + 4$

switch x & y

$x = 5y^3 + 4$

solve for y.

$x - 4 = 5y^3$

$\frac{x-4}{5} = y^3$

$y = \sqrt[3]{\frac{x-4}{5}}$

$f^{-1}(x) = \sqrt[3]{\frac{x-4}{5}}$

Problem 20 (Logarithmic equations). Solve the equation

$$\log_5(x-10) = 2$$

$5^2 = (x-10)$

$25 = x-10 \quad /+10$

$35 = x$

check:

$\log_5(35-10) = \log_5 25 = \boxed{2}$ ✓

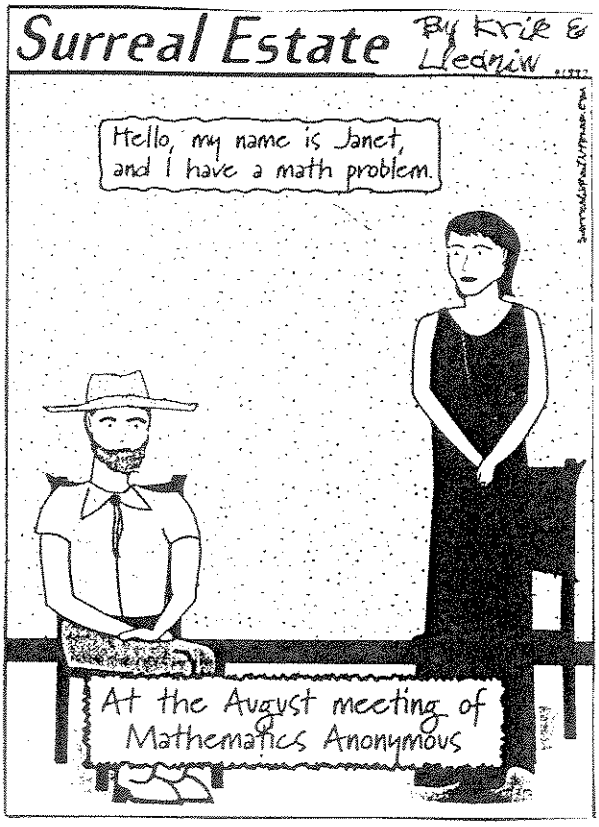
$5^{\square} = 25$

$5^2 = 25$

$x = \underline{35}$

If you like separate this page, take a note of your answers on this page, and compare them with those on the answer sheet.

I very much enjoyed teaching this class and hope you found it a worthwhile experience. I hope you are successful in your future endeavors.


Surreal Estate
By Erik & Liedgiw
Hello, my name is Janet, and I have a math problem.
At the August meeting of Mathematics Anonymous