HW # 10 Key (Graded Problems)

~~5.1 # 4, 8, 20, 32, 41~~ $\boxed{5.2}$ # 4, 9, 14, 20, 22, 36 (EC)

5.1 # 4 | shirt comes in 12 colors
Male or female version
3 sizes

How many types of shirt?

$12(2)(3) = \boxed{72}$

5.1 # 8 / How many different 3-letter initials, w/ no repeats, can people have?

$26(25)(24) = \boxed{15600}$

5.1 # 20 | How many positive integers less than 1000
i.e. from 1, 2, ..., 999

(a) divisible by 7? $\left\lfloor \frac{999}{7} \right\rfloor = \boxed{142}$

(b) divisible by 7 but not 11?
\# divisible by both 7 & 11 $= \left\lfloor \frac{999}{77} \right\rfloor = 12$

$\Rightarrow 142 - 12 = \boxed{130}$

(c) divisible by both 7 & 11 $= \boxed{12}$ (above)

(d) divisible by either 7 or 11 $= \left\lfloor \frac{999}{7} \right\rfloor + \left\lfloor \frac{999}{11} \right\rfloor - 12$

$= 142 + 90 - 12 = \boxed{220}$

(e) divisible by exactly one of 7 or 11 $= 220 - 12 = \boxed{208}$

(f) divisible by neither 7 nor 11 $= 999 - 220 = \boxed{779}$

5.1 #20 (cont)

(g) have distinct digits = ?

1-digit : 9

2-digit : $\underline{9}\cdot\underline{9} = 81$

3-digit : $\underline{9}\cdot\underline{9}\cdot\underline{8} = 648$

$648+81+9 = \boxed{738}$

(h) have distinct digits and are even?

\# odds w/ distinct digits:

1-digit: 5

2-digit: $\underline{8}\cdot\underline{5} = 40$

3-digit: $\underline{8}\cdot\underline{8}\cdot\underline{5} = 320$

$\Rightarrow 320+40+5 = 365$ odd #s w/ distinct digits

$\Rightarrow 738-365 = \boxed{373}$ even #s w/ distinct digits

5.1 # 32 How many fns from A to B $\ni |A| = 10$

and (a) $|B| = 2$ $\quad \underset{a_1}{\underline{2}}\cdot\underset{a_2}{\underline{2}}\cdot\underset{a_3}{\underline{2}}\cdot\underset{a_4}{\underline{2}}\cdot\underset{a_5}{\underline{2}}\cdot\underset{a_6}{\underline{2}}\cdot\underset{a_7}{\underline{2}}\cdot\underset{a_8}{\underline{2}}\cdot\underset{a_9}{\underline{2}}\cdot\underset{a_{10}}{\underline{2}} \Rightarrow \boxed{2^{10}}$

(b) $|B| = 3$ $\quad \boxed{3^{10} \text{ fns}}$

(c) $|B| = 4$ $\quad \boxed{4^{10} \text{ fns}}$

(d) $|B| = 5$ $\quad \boxed{5^{10} \text{ fns}}$

5.1 # 41 How many ways to photograph 6 people in a row, including bride and groom.

(a) bride must be next to groom

$\underline{B}\ \underline{G}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

or $\underline{4}\ \underline{B}\ \underline{G}\ \underline{3}\ \underline{2}\ \underline{1}$

or $\underline{4}\ \underline{3}\ \underline{B}\ \underline{G}\ \underline{2}\ \underline{1}$

or $\underline{4}\ \underline{3}\ \underline{2}\ \underline{B}\ \underline{G}\ \underline{1}$

or $\underline{4}\ \underline{3}\ \underline{2}\ \underline{1}\ \underline{B}\ \underline{G}$

5.1 # 41 (cont)

i.e. there are 5 choices of where bride is & that fixes the groom & for each of those 5 choices, there are 4! ways to arrange others. And all those choices can be doubled since BG can be replaced w/ GB.

$\Rightarrow \quad 2(5)(4!) = \boxed{240}$

(b) bride not next to groom.

ex

B 4 G 3 2 1

B 4 3 G 2 1

B 4 3 2 G 1

B 4 3 2 1 G

if B is ~~place~~ place 1, 4(4!) ways
" B " place 2, 3(4!) "
" " " " 3, 3(4!) "
" " " " 4, 3(4!) "
" " " " 5, 3(4!) "
" " " " 6, 4(4!) "

$\Rightarrow$ # ways =
$2(4)(4!) + 4(3)(4!)$
$= 5(4)(4!)$
$= \boxed{480}$

(c) bride is to left of groom

B in place 1, groom has 5 choices 5(4!)
B in place 2, " " 4 choices 4(4!)
" " " 3, " " 3 choices 3(4!)
" " " 4, " " 2 " 2(4!)
" " " 5, " " 1 " 1(4!)

$\Rightarrow$ # ways $= (5+4+3+2+1)4! = 15(4!) = \boxed{360}$

5.2 #4) bowl has 5R 10B balls; select balls at random

(a) how many balls must be selected to ensure at least 3 balls of same color?

$\boxed{5}$ worst case scenario ex, pull RBRBR

(b) how many balls must be selected to ensure at least 3 blue balls

worst case scenario RRRRRBBB

$\boxed{8}$

5.2 #9) minimum # of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee at least 100 from same state? we want $\left\lceil \frac{N}{k} \right\rceil \geq 100 \Rightarrow k = 50$

$$\frac{N}{50} + 1 > \left\lceil \frac{N}{50} \right\rceil \geq 100 \iff \frac{N}{50} > 99$$

$$N > 4950$$

$\Rightarrow N = \boxed{4951, \text{ at least}}$

5.2 #14) (a) Show if 7 integers selected from first 10 +ve integers, $\exists$ at least 2 pairs w/ sum 11.

$\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ $\left\lceil \frac{7}{5} \right\rceil = 2$

$\{1,10\}$ $\{2,9\}$ $\{3,8\}$ $\{4,7\}$ $\{5,6\}$ pigeonhole principle

$\Rightarrow$ we have at least one box w/ 2 & since we're selecting 7 #s, there must be one more dropped into a second box, causing another pair that adds to 11.

5.2 #14 (cont)

(b) Is (a) true if 6 are selected rather than 7?

No, if we end up choosing #s 1, 2, 3, 4, 5, 6 then we only have one pair that adds to 11.

5.2 #20) Find increasing subsequence of max length + a decreasing subsequence of max length in

22, 5, 7, 2, 23, 10, 15, 21, 3, 17

note: we're guaranteed one inc. or dec. sequence of length 4

increasing:

5, 7, 10, 15
5, 7, 10, 21
5, 7, 10, 17
5, 7, 15, 17
5, 7, 15, 21
7, 10, 15, 21
5, 10, 15, 21

decreasing:

22, 10, 3
22, 15, 3
22, 21, 17
22, 5, 2
22, 7, 2
22, 5, 3
22, 7, 3

23, 10, 3
23, 15, 3
23, 21, 3
23, 21, 17

none of length 4

5.2 #22) Show if ∃ 101 people of different heights standing in a line, it's possible to find 11 people in the order they're standing that are either increasing or decreasing in height.

$101 = 10^2 + 1 \Rightarrow$ by Thm 3, we have a subsequence of length 11 that's increasing or decreasing

HW10-5

5.2 #36 (Extra Credit)

Prove At a party, where $\exists$ at least 2 people, $\exists$ 2 people who know the same # of people there.

Pf let $n =$ # people at party, $n \geq 2$.

let $f(x) =$ # of people at the party who person x knows

(Assume if x knows y, then y knows x.)

Then $f: A \to B \Rightarrow |A| = n$ and $B = \{0, 1, 2, \dots, n-1\}$

$\Rightarrow |B| = n$

but if $f(x) = 0$, then x knows no-one

and if $f(x) = n-1$, then x knows everyone & thus there is no person who knows no-one

i.e. $f(x) = 0 \Rightarrow \nexists\ y \ni f(y) = n-1$

and $f(x) = n-1 \Rightarrow \nexists\ y \ni f(y) = 0$

In other words, we can have only 0 or $n-1$ (not both) as possible outputs for f.

$\Rightarrow |B| = n-1 \Rightarrow f: A \to B$ is a fn from n elements to $n-1$ elements $\Rightarrow$ at least two inputs get mapped to same output, i.e. at least 2 people know same # of others.