HW #12 Key (for Graded Problems)

$\boxed{5.5}$ # 2, 14, 22, 30, 43 $\boxed{6.1}$ # 7, 12, 18, 34, 41

5.5 #2) # ways 5 elements selected (in order) from set w/ 5 elements when repetition allowed

$= 5^5 = 3125$

5.5 #14) How many solns to $x_1 + x_2 + x_3 + x_4 = 17$ $x_1, x_2, x_3, x_4 \in W$?

${}_{4+17-1}C_{17} = {}_{20}C_{17} = \frac{20!}{17!\,3!} = \frac{20(19)(\cancel{18}^{3})}{\cancel{6}}$

$= 1140$

5.5 #22) # ways to distribute 12 indistinguishable balls into 6 distinguishable bins

= # ways to choose 6 bins from 12 bins w/ repetition

$= {}_{6+12-1}C_{12} = {}_{17}C_{12} = \frac{17!}{12!\,5!} = 6188$

5.5 #30) # strings from MISSISSIPPI (using all letters)

11 letters total	
1	M
4	I
4	S
2	P

$= \frac{11!}{1!\,4!\,4!\,2!} = 34650$

5.5 #43) # ways to deal hands of 5 cards to each of 6 players from a deck of 48 different cards $= \frac{{}_{48}P_{30}}{5!\,5!\,5!\,5!\,5!\,5!} \simeq 6.49 \times 10^{32}$

deal 30 cards total

$\Rightarrow {}_{48}P_{30}$

6.1 #7 $P(HHHHHH) = \frac{1}{2^6} = \frac{1}{64}$

6.1 #12 P(5-card poker hand has exactly one ace)

A $\underline{48}$ $\underline{47}$ $\underline{46}$ $\underline{45}$ $\Rightarrow$ ${}_{48}C_4$ ways to have one distinct Ace
$\Rightarrow$ $4({}_{48}C_4)$ ways to have exactly one Ace (since $\exists$ 4 Aces)

and $\exists$ ${}_{52}C_5$ ways to get a poker hand

$\Rightarrow$ Prob $= \frac{4({}_{48}C_4)}{{}_{52}C_5} \simeq 0.30$

6.1 #18 P(5-card poker hand contains straight flush)
(straight flush = 5 cards of same suit of consecutive kinds)

A2345, 23456, 34567, 45678, 56789, 678910, 78910J, 8910JQ, 910JQK, 10JQKA $\Rightarrow$ 10 straight flushes per suit

$\Rightarrow$ 40 straight flushes

$\Rightarrow$ Prob $= \frac{40}{{}_{52}C_5} = \frac{1}{64974}$

6.1 #34 drawing of 50 people
probability of
B1
C2
J3
R4
winners?

(a) no one can win more than one prize
(b) winning more than one prize is allowed

(note: (a) no replacement (b) replacement)

6.1#34 (cont) There are $_{50}P_4$ equally likely outcomes of winners.

(a) $P(B1, C2, J3, R4) = \frac{1}{_{50}P_4} = \frac{1}{5{,}527{,}200}$

(b) If repetition is allowed, there are 50^4 equally likely outcomes of winners.

$\Rightarrow P(B1, C2, J3, R4) = \frac{1}{50^4} = \frac{1}{6{,}250{,}000}$

6.1#41 (a) P(at least one 6 on 4 rolls of a die)

$= 1 - P(\text{no 6 in 4 rolls})$

each roll is not influenced by previous rolls

For one roll,

$P(6) = \frac{1}{6}$

$P(\text{not } 6) = \frac{5}{6}$

$= 1 - [P(\text{not } 6)]^4$

$= 1 - \left(\frac{5}{6}\right)^4 = \frac{671}{1296} \simeq 0.518$

(b) P(66 at least once in 24 rolls of 2 die)

$= 1 - P(\text{no 66 in 24 rolls})$

$= 1 - (P(\text{not } 66))^{24}$

$= 1 - \left(\frac{35}{36}\right)^{24} \simeq 0.491 < 0.5$

For 1 roll of 2 dice,

$P(66) = \frac{1}{36}$

$P(\text{not } 66) = \frac{35}{36}$

(c) (a) more likely than (b)