HW #13 Key (for Graded Problems)

6.2 #1, 2, 6, 12, 20, 24, 34
6.3 # 2, 3, 6 (16 for EC)

6.2 #1) H 3 times as likely as tails

$P(H)+P(T)=1$ Let $P(T)=x$ then $P(H)=3x$

and $3x+x=1 \Rightarrow x=1/4$

$\Rightarrow P(H)=\frac{3}{4}$ and $P(T)=\frac{1}{4}$

6.2 #2) $P(3)=2x$ $P(1)=P(2)=P(4)=P(5)=P(6)=x$

$P(1)+P(2)+P(3)+P(4)+P(5)+P(6)=1$

$\Leftrightarrow 5x+2x=1 \Rightarrow x=1/7$

$\Rightarrow P(3)=2/7$ $P(1)=P(2)=P(4)=P(5)=P(6)=1/7$

6.2 #6) select a permutation from $\{1,2,3\}$ $\Rightarrow \exists\ {}_3P_3 = 3! = 6$ permutations total

(a) $P(1 \text{ precedes } 3) = \frac{3}{6} = \frac{1}{2}$

1 _ _ 2 choices
_ 1 3 1 choice

(b) $P(3 \text{ precedes } 1) = \frac{1}{2}$
(same logic as for (a))

(c) $P(3 \text{ precedes } 1 \text{ and } 3 \text{ precedes } 2)$
$= \frac{2}{6} = \frac{1}{3}$

3 _ _
2 choices

6.2 #12) $P(E)=0.8$ $P(F)=0.6$ show $P(E\cup F)\geq 0.8$ and $P(EF)\geq 0.4$.

$P(E\cup F)=P(E)+P(F)-P(EF)=0.8+0.6-P(EF)$

6.2 #12 (cont)

we know $P(EF) < P(F)$ [since $E \neq F$ because $P(E) \neq P(F)$ $\Rightarrow EF \subset F$]

and $P(F) = 0.6$

$\Leftrightarrow P(EF) < 0.6$

$\Leftrightarrow 0 < 0.6 - P(EF)$

$\Rightarrow P(E \cup F) = 0.8 + (0.6 - P(EF))$

$> 0.8 + 0 = 0.8$ //

6.2 #20

smallest # people needed so probability that at least one of them has bday today exceeds 1/2

(assume 366 equally likely bdays + bdays are indep.)

$P(\text{none has bday today}) = \left(\frac{365}{366}\right)^n$

want smallest $n \ni 1 - \left(\frac{365}{366}\right)^n \geq \frac{1}{2}$

$\Leftrightarrow \frac{1}{2} \geq \left(\frac{365}{366}\right)^n$

$\ln\left(\frac{1}{2}\right) \geq n \ln\left(\frac{365}{366}\right)$

$n \leq \frac{\ln\left(\frac{1}{2}\right)}{\ln\left(\frac{365}{366}\right)} \simeq 253$

but $\left(\frac{365}{366}\right)^{254} \approx 0.4991 \Rightarrow n = 254$

6.2 #24) out of 5 tosses T_1 = 1st toss is tails

$P(4H \mid T_1) = \frac{1}{2^4} = \frac{1}{16}$ T_1 _ _ _ _

6.2 #34) prob of success $= p$, n indep trials

(a) $P(0 \text{ successes}) = {}_nC_0\, p^0 (1-p)^n = (1-p)^n$

(b) $P(\text{at least one success}) = 1 - P(\text{no successes})$
$= 1 - (1-p)^n$

(c) $P(\text{at most one success}) = P(0 \text{ successes}) + P(1 \text{ success})$

$= (1-p)^n + {}_nC_1\, p^1 (1-p)^{n-1} = (1-p)^n + np(1-p)^{n-1}$

(d) $P(\text{at least 2 successes})$
$= 1 - P(0 \text{ successes}) - P(1 \text{ success})$
$= 1 - (1-p)^n - np(1-p)^{n-1}$

6.3 #2) E, F in S $P(E) = 2/3$, $P(F) = 3/4$,
$\Rightarrow P(\bar{F}) = 1/4$
$P(\bar{E}) = 1/3$
$P(F \mid E) = 5/8$, $P(E \mid F) = ?$

$P(E \mid F) = \frac{P(F \mid E)P(E)}{P(F \mid E)P(E) + P(F \mid \bar{E})P(\bar{E})}$

$= \frac{\frac{5}{8}\left(\frac{2}{3}\right)}{\frac{5}{8}\left(\frac{2}{3}\right) + 1\left(\frac{1}{3}\right)} = \frac{\frac{5}{12}}{\frac{5}{12} + \frac{1}{3}}$

$= \frac{5}{5+4} = \frac{5}{9}$

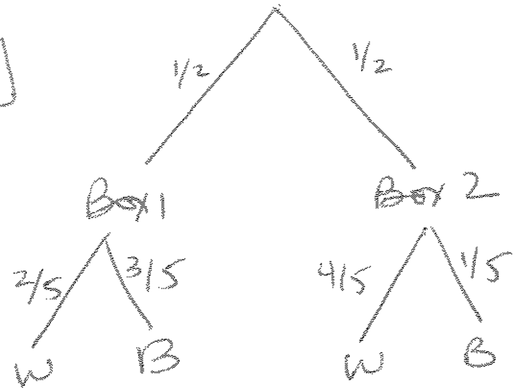
$P(F \mid E) = \frac{P(EF)}{P(E)} \Rightarrow P(EF) = \frac{2}{3}\left(\frac{5}{8}\right) = \frac{5}{12}$

$P(F \mid \bar{E}) = \frac{P(F\bar{E})}{P(\bar{E})} = \frac{1/3}{1/3} = 1$

and $P(F\bar{E}) = P(F) - P(FE)$
$= \frac{3}{4} - \frac{5}{12} = \frac{4}{12} = \frac{1}{3}$

HW13-3

6.3 #3

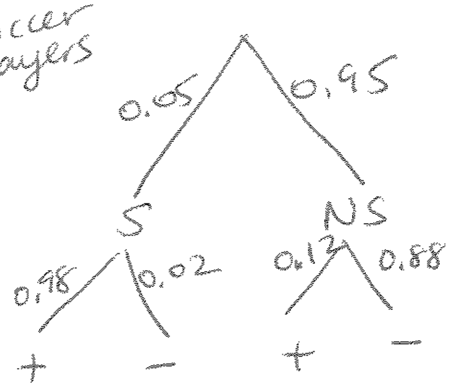


$P(\text{Box }1 \mid B) = ?$

$$P(\text{Box }1 \mid B) = \frac{P(B \mid \text{Box }1)\,P(\text{Box }1)}{P(B \mid \text{Box }1)\,P(\text{Box }1) + P(B \mid \text{Box }2)\,P(\text{Box }2)}$$

$$= \frac{\frac{3}{5}\left(\frac{1}{2}\right)}{\frac{3}{5}\left(\frac{1}{2}\right) + \frac{1}{5}\left(\frac{1}{2}\right)} = \frac{3}{3+1} = \frac{3}{4}$$

6.3 # 6 soccer players



$P(S \mid +) = ?$

$$P(S \mid +) = \frac{P(+ \mid S)\,P(S)}{P(+ \mid S)\,P(S) + P(+ \mid NS)\,P(NS)}$$

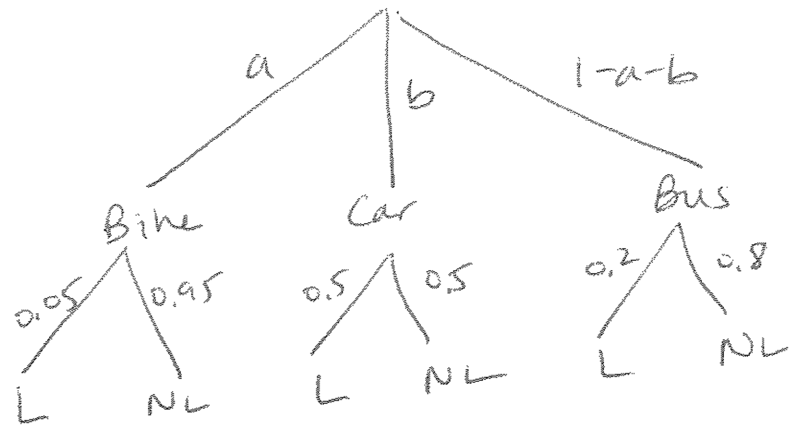
$$= \frac{0.98(0.05)}{0.98(0.05) + 0.12(0.95)}$$

$$= \frac{0.049}{0.049 + 0.114} = \frac{0.049}{0.163}$$

$$\cong 0.3006$$

$\Rightarrow$ about 30%

6.3 #16 (Extra Credit)



$P(\text{car} \mid L) = ?$

$$P(\text{car} \mid L) = \frac{P(L \mid \text{car})\, P(\text{car})}{P(L \mid \text{car}) P(\text{car}) + P(L \mid \text{Bike}) P(\text{Bike}) + P(L \mid \text{Bus}) P(\text{Bus})}$$

$$= \frac{0.5b}{0.5b + 0.05a + 0.2(1-a-b)} = \frac{0.5b}{0.2 + 0.3b - 0.15a}$$

(a) assume $a = b = \frac{1}{3}$

$$\Rightarrow P(\text{car} \mid L) = \frac{0.5(\frac{1}{3})}{0.2 + 0.3(\frac{1}{3}) - 0.15(\frac{1}{3})} = \frac{0.5}{0.6 + 0.3 - 0.15}$$

$$= \frac{50}{75} = \frac{2}{3}$$

(b) assume $a = 0.6$
$b = 0.3$

$$\Rightarrow P(\text{car} \mid L) = \frac{0.5(0.3)}{0.2 + 0.3(0.3) - 0.15(0.6)} = \frac{0.15}{0.2} = \frac{15}{20} = \frac{3}{4}$$