Graded Problems

2.1 # 8, 10, 34 | 2.2 # 17, 26, 48 | 2.3 # 2, 18, 34, 38

M2200
HW#4

2.1 #8) (a) $\phi \in \{\phi\}$ T (b) $\phi \in \{\phi, \{\phi\}\}$ T

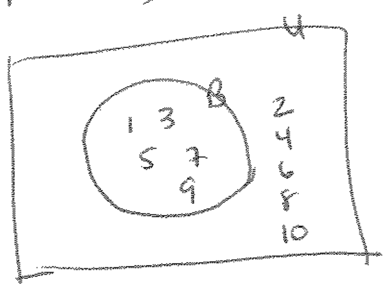
(c) $\{\phi\} \in \{\phi\}$ F (d) $\{\phi\} \in \{\{\phi\}\}$ T

(e) $\{\phi\} \subset \{\phi, \{\phi\}\}$ T (f) $\{\{\phi\}\} \subset \{\phi, \{\phi\}\}$ T

(g) $\{\{\phi\}\} \subset \{\{\phi\}, \{\phi\}\}$ F

#10 Venn diagram: illustrate subset of odd integers in set of all $\mathbb{Z}^+$ not exceeding 10

let $U = \{x \mid x \in \mathbb{Z}, 0 < x \leq 10\}$ $B = \{x \mid x = 2k+1, k \in \mathbb{Z}\}$



#34 Translate to English.

(a) $\exists x \in \mathbb{R} (x^3 = -1)$ "There is a real # such that the number cubed is -1."

(b) $\exists x \in \mathbb{Z} (x+1 > x)$ "There is an integer x such that one more than x is bigger than itself."

(c) $\forall x \in \mathbb{Z} (x-1 \in \mathbb{Z})$ "If x is an integer, so is $x-1$."

(d) $\forall x \in \mathbb{Z} (x^2 \in \mathbb{Z})$ "If x is an integer, so is x^2."

2.2 **#17** Show $\overline{A\cap B\cap C} = \bar{A}\cup\bar{B}\cup\bar{C}$

(a) by proving each side is a subset of the other side

(b) by a membership table.

(a) Pf: assume $x\in\overline{A\cap B\cap C}$. Then

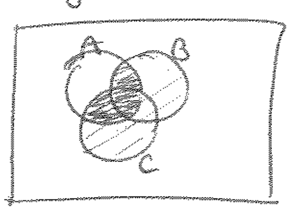
$$\iff x\notin A\cap B\cap C \equiv x\notin A \vee x\notin B \vee x\notin C$$
$$\equiv x\in\bar{A} \vee x\in\bar{B} \vee x\in\bar{C}$$
$$\equiv x\in(\bar{A}\cup\bar{B}\cup\bar{C})$$

(b)

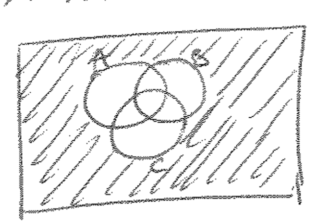
A	B	C	$A\cap B\cap C$	$-(A\cap B\cap C)$	$\bar{A}\cup\bar{B}\cup\bar{C}$
1	1	1	1	0	0
1	1	0	0	1	1
1	0	1	0	1	1
1	0	0	0	1	1
0	1	1	0	1	1
0	1	0	0	1	1
0	0	1	0	1	1
0	0	0	0	1	1

#26 Draw Venn diagrams

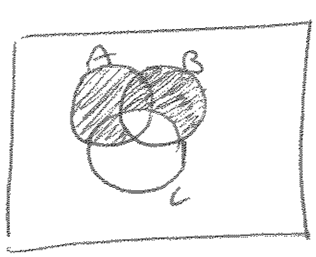
(a) $A\cap(B\cup C)$



(b) $\bar{A}\cap\bar{B}\cap\bar{C}$



(c) $(A-B)\cup(A-C)\cup(B-C)$



2.2 #48) Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$, $i \in \mathbb{Z}^+$.

(a) $A_i = \{i, i+1, i+2, \ldots\}$ $\qquad A_1 = \{1,2,3,\ldots\}$
$\qquad A_2 = \{2,3,4,\ldots\}$

$\bigcap_{i=1}^{\infty} A_i = \emptyset \qquad \bigcup_{i=1}^{\infty} A_i = \{1,2,3,\ldots\} = A_1$

(b) $A_i = \{0, i\}$ $\qquad A_1 = \{0,1\} \quad A_2 = \{0,2\} \quad A_3 = \{0,3\}$

$\bigcap_{i=1}^{\infty} A_i = \{0\} \qquad \bigcup_{i=1}^{\infty} A_i = \{0,1,2,3,\ldots\}$

(c) $A_i = (0, i)$ $\qquad$ (i.e. the interval from 0 to i, noninclusive)

$\bigcap_{i=1}^{\infty} A_i = (0,1) \cap (0,2) \cap (0,3) \ldots = (0,1) = A_1$

$\bigcup_{i=1}^{\infty} A_i = (0,1) \cup (0,2) \cup (0,3) \ldots$ $\quad$ (notice $(0,1) \cup (0,2) = (0,2)$
and $(0,2) \cup (0,3) = (0,3)$)
$= (0, \infty)$

(d) $A_i = (i, \infty)$ $\qquad$ (again an open interval of $\mathbb{R}$).

$\bigcap_{i=1}^{\infty} A_i = (1,\infty) \cap (2,\infty) \cap (3,\infty) \cap \ldots = \emptyset$

$\bigcup_{i=1}^{\infty} A_i = (1, \infty) = A_1$

2.3 #2) Is f a function from $\mathbb{Z}$ to $\mathbb{R}$?

(a) $f(n) = \pm n$ $\qquad$ No, because each input maps to more than one output

(b) $f(n) = \sqrt{n^2+1}$ $\qquad$ Yes, each input maps to exactly one output

(c) $f(n) = \dfrac{1}{n^2-4}$ $\qquad$ No, because for $n = \pm 2$, there is no output

$\boxed{2.3}$ #18 $f: \mathbb{R} \to \mathbb{R}$ bijection?

(a) $f(x) = -3x + 4$

1-1 yes

onto yes $\Rightarrow$ yes bijection

(b) $f(x) = -3x^2 + 7$

1-1 no

because if $f(x) = f(y)$, then we get $-3x^2 + 7 = -3y^2 + 7$

$$x^2 = y^2$$
$$x = \pm y$$

$\Rightarrow$ not a bijection

onto no

To be onto, we'd need an x to exist $\forall r \in \mathbb{R}$ (codomain) $\ni f(x) = r$.

But if $r = 10$, then

$$-3x^2 + 7 = 10$$
$$-3x^2 = 3$$
$$x^2 = -1 \Rightarrow \text{N.S.}$$

i.e. there is no $x \in \mathbb{R} \ni f(x) = 10$.

(c) $f(x) = \frac{(x+1)}{(x+2)}$

1-1 $f(x) = f(y)$

$$\frac{x+1}{x+2} = \frac{y+1}{y+2}$$

$$\cancel{xy} + 2x + y + \cancel{2} = \cancel{xy} + x + 2y + \cancel{2}$$

$$2x + y = x + 2y$$

$$x = y \checkmark$$

yes

onto Let $r \in \mathbb{R}$.

Then $r = \frac{x+1}{x+2}$

$$rx + 2r = x + 1$$
$$x(r-1) = 1 - 2r$$
$$x = \frac{1-2r}{r-1}$$

However, if $r = 1$, then x DNE to map to 1.

most important [And if $x = -2$, it doesn't map to anything

$\Rightarrow$ $f(x)$ not a bijection

$\boxed{2.3}$ #18) (cont)

(d) $f(x)=x^5+1$

1-1 $f(x)=f(y)$

$x^5+1=y^5+1$

$x=y$ ✓

yes

onto let $r\in\mathbb{R}$.

$r=x^5+1$

$x=\sqrt[5]{r-1}$ which is well-defined $\forall r\in\mathbb{R}$

yes

$\Rightarrow f(x)$ is bijection

$\boxed{2.3}$ #34) $f(x)=ax+b$ $g(x)=cx+d$ $a,b,c,d\in\mathbb{R}$ (constants)

what condition is true for a,b,c,d $\ni f\circ g=g\circ f$?

$f(g(x))=f(cx+d)=a(cx+d)+b=acx+ad+b$

$g(f(x))=g(ax+b)=c(ax+b)+d=acx+bc+d$

we want $acx+ad+b=acx+bc+d$

$\boxed{ad+b=bc+d}$

#38) $f:\mathbb{R}\to\mathbb{R}$ $f(x)=x^2$.

☆ Note: The book takes the liberty here to "twist" the meaning of the inverse notation! Please notice that $f^{-1}(x)$ DNE, for this domain & codomain. These questions are using the f^{-1} notation to ask for the pre-image.

2.3

#38 (cont)

(a) $f^{-1}(\{1\})$ This is asking for the set of inputs that get mapped to the given set of outputs, namely $\{1\}$, under $f(x)$.

$\Leftrightarrow$ $f(?) = 1$ $\Leftrightarrow$ $x^2 = 1$ $\Leftrightarrow$ $x = \pm 1$

$\Rightarrow$ preimage of $\{1\} = \{-1, 1\}$

(b) $f^{-1}(\{x \mid 0 < x < 1\}) = ?$ is asking for the pre-image of $\{x \mid 0 < x < 1\}$

$\Leftrightarrow$ $x^2 = y$ and we want $0 < y < 1$

$\Leftrightarrow$ $0 < x^2 < 1$ $\Leftrightarrow$ $|x| < 1$

$\Rightarrow$ pre-image of $\{x \mid 0 < x < 1\} = \{x \mid |x| < 1\}$

(c) $f^{-1}(\{x \mid x > 4\})$ = pre-image of $\{x \mid x > 4\}$

$\Leftrightarrow$ $x^2 = y$ $\ni$ $y > 4$

$\Leftrightarrow$ $x^2 > 4$ $\Leftrightarrow$ $x > 2$ or $x < -2$

$\Rightarrow$ preimage of $\{x \mid x > 4\} = \{x \mid x > 2 \cup x < -2\}$