4.3 Induction & Recursion

Recursively Defined Fns

$f: \mathbb{W} \to A$ (A can be any subset of $\mathbb{R}$)

① Specify fn value at $n=0$.
② Give rule for finding next f-value based on previous f-value(s).

Ex 1 Find $f(1), f(2), f(3), f(4), f(5)$ if $f(0)=3$ and

(a) $f(n+1) = 3f(n) + 7$

(b) $f(n+1) = 3^{f(n)/3}$

4.3 (cont)

Ex 2 Are these valid recursive fns f from $\mathbb{W}$ to $\mathbb{Z}$? If f well-defined, find $f(n)$ for any $n \in \mathbb{W}$ + prove your formula is valid.

(a) $f(0)=0$, $f(n)=2f(n-2)$ $\forall n \geq 1$

(b) $f(0)=1$, $f(1)=2$, $f(n)=2f(n-2)$ $\forall n \geq 2$

4.3 (cont)

Ex 2 (cont)

(c) $f(0)=1,\ f(n)=f(n-1)-1 \quad \forall\, n \geq 1$

4.3 (cont)

Ex 3 Find recursive formula for number of diagonals in a polygon (n-gon) $\forall\ n=3,4,5,\ldots$ Then, find a direct formula & prove your claim.

Defn 1 Fibonacci numbers $f_0, f_1, f_2, \ldots$ are defined by eqns $f_0 = 0, f_1 = 1$, and $f_n = f_{n-1} + f_{n-2}$, $\forall\, n = 2, 3, 4, \ldots$

Ex 4 Show $f_{n+1} f_{n-1} - f_n^{\,2} = (-1)^n \quad \forall\, n \in \mathbb{Z}^+$.

Ex 5 Use result above to prove
$$f_0 f_1 + f_1 f_2 + \ldots + f_{2n-1} f_{2n} = f_{2n}^{\,2} \quad n \in \mathbb{Z}^+.$$