

## M2200 Midterm 1 Review

- (1) Prove  $(\bar{q} \wedge (p \rightarrow q)) \rightarrow \bar{p}$  is a tautology.
- (2) Use quantifiers. "There is a building on the campus of some college in the US in which every room is painted white."  
What is the negation?
- (3) What are logical conclusions? "All foods that are healthy don't taste good."  
"Almonds are healthy to eat." "Carol only eats what tastes good." "Carol is allergic to almonds, so she doesn't eat them." "Carol is not healthy to eat."
- (4) (pg 20 #60)  
It's a boolean logic problem.
- (5) List elements of the set  $\{x \mid x \in \mathbb{R}, x^2 = 1 \text{ or } x^2 = 5\}$
- (6) Show  $(B-A) \cup (C-A) = (B \cup C) - A$
- (7) What can you say about  $A, B$  if  
(a)  $A - B = A$                       (b)  $A - B = B - A$
- (8) Let  $A_i = \{\dots, -2, -1, 0, 1, \dots, i\}$ , find  $\bigcup_{i=1}^{\infty} A_i$  and  $\bigcap_{i=1}^{\infty} A_i$ .

(9) Is  $f: \mathbb{R} \rightarrow \mathbb{R}$  H? onto? for

(a)  $f(x) = 2x + 10$

(b)  $f(x) = x^4 + 1$

(c)  $f(x) = \frac{x^3 + 1}{x^3 - 2}$

If  $f$  is bijective,  
find  $f^{-1}(x)$ .

(10) Prove that  $\forall x \in \mathbb{R}, x \geq 0 \quad \lfloor \sqrt{\lfloor x \rfloor} \rfloor = \lfloor \sqrt{x} \rfloor$ .

(11) Prove if  $n$  is odd, then  $n^2$  is odd.

(12) Prove  $\sqrt{5}$  is irrational.

(13) Prove if  $n^2$  is divisible by 3, then  $n$  is divisible by 3.

(14) Prove:  $\lfloor 2x \rfloor = \lfloor x \rfloor + \lfloor x + \frac{1}{2} \rfloor$

(15) Prove  $n^2 \geq 2n \quad \forall n \in \mathbb{N}$ .

(16) Prove between any 2 distinct rational numbers, there exists a rational number.