Chapter 17 Solutions

17.1. $s/\sqrt{n} = 21.88/\sqrt{20} \doteq 4.8925$ minutes.

17.2. The mean is $\bar{x} = 251$ ms. Because $s/\sqrt{n} = s/\sqrt{12} = 45$ ms, we have $s \doteq 155.88$ ms.

17.3. **(a)** $t^* = 2.015$ for df $= 5$ and $\alpha = 0.05$. **(b)** $t^* = 2.518$ for df $= 21$ and $\alpha = 0.01$.

17.4. Use df $= 24$. **(a)** $t^* = 2.064$ for $\alpha = 0.025$. **(b)** $t^* = 0.685$ for $\alpha = 0.25$.

17.5. **(a)** Use df $= 9$ and $\alpha = 0.025$: $t^* = 2.262$. **(b)** Use df $= 19$ and $\alpha = 0.005$: $t^* = 2.8$
(c) Use df $= 6$ and $\alpha = 0.05$: $t^* = 1.943$.

17.6. **(a)** The stemplot shows a slight skew to the right, but not so strong that it would invalidate the t procedures. **(b)** With $\bar{x} = 18.66$ and $s \doteq 10.2768$, and $t^* = 2.093$ (df $= 19$), the 95% confidence interval for μ is

$$18.66 \pm 2.093\left(\frac{10.2768}{\sqrt{20}}\right) = 18.66 \pm 4.8096 = 13.8504 \text{ to } 23.4696.$$

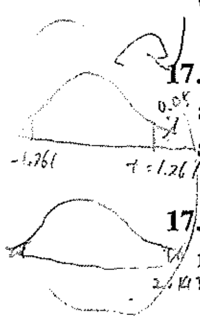
0	67
1	00
1	67
2	01
2	57
3	00
3	78

17.7. STATE: What is the mean percent μ of nitrogen in ancient air?
PLAN: We will estimate μ with a 90% confidence interval.
SOLVE: We are told to view the observations as an SRS. A stemplot shows some left-skewness; however, for such a small sample, the data are not unreasonably skewed. There are no outliers. With $\bar{x} = 59.5889\%$ and $s = 6.2553\%$ nitrogen, and $t^* = 1.860$ (df $= 8$), the 90% confidence interval for μ is

$$59.5889 \pm 1.860\left(\frac{6.2553}{\sqrt{9}}\right) = 59.5889 \pm 3.8783 = 55.71\% \text{ to } 63.47\%.$$

4	9
5	1
5	
5	4
5	
5	
6	0
6	33
6	44

CONCLUDE: We are 90% confident that the mean percent of nitrogen in ancient air is between 55.71% and 63.47%.



17.8. **(a)** df $= 14$. **(b)** $t = 1.82$ is between 1.761 and 2.145, for which the one-sided P-valu are 0.05 and 0.025, respectively. (Software reports that $P = 0.0451$.) **(c)** $t = 1.82$ is significant at $\alpha = 0.05$ but not at $\alpha = 0.01$.

17.9. **(a)** df $= 24$. **(b)** $t = 1.12$ is between 1.059 and 1.318, so $0.20 < P < 0.30$. (Softwar reports that $P = 0.2738$.) **(c)** $t = 1.12$ is not significant at either $\alpha = 0.10$ or $\alpha = 0.05$.

17.10. STATE: Is there evidence that the percent of nitrogen in ancient air was different fro the present 78.1%?
PLAN: We test H_0: $\mu = 78.1\%$ vs. H_a: $\mu \neq 78.1\%$. We use a two-sided alternative beca prior to seeing the data, we had no reason to believe that the percent of nitrogen in ancie air would be higher or lower.
SOLVE: We addressed the conditions for inference in Exercise 17.7. In that solution, we found $\bar{x} = 59.5889\%$ and $s = 6.2553\%$ nitrogen, so $t = \frac{59.5889-78.1}{6.2553/\sqrt{9}} \doteq -8.88$. For df $=$

this is beyond anything shown in Table C, so $P < 0.001$ (software gives $P = 0.00002$).
CONCLUDE: We have very strong evidence ($P < 0.001$) that Cretaceous air contained less nitrogen than modern air.

17.11. PLAN: Take μ to be the mean difference (monkey call minus pure tone) in firing rate. We test H_0: $\mu = 0$ vs. H_a: $\mu > 0$, using a one-sided alternative because the researchers suspect a stronger response to the monkey calls.
SOLVE: We must assume that the monkeys can be regarded as an SRS. For each monkey, we compute the call minus pure tone differences; a stemplot of these differences shows no outliers or deviations from Normality. The mean and standard deviation are $\bar{x} \doteq 70.378$ and $s \doteq 88.447$ spikes/second, so $t = \frac{\bar{x}-0}{s/\sqrt{37}} \doteq 4.84$ with df $= 36$. This has a very small P-value: $P < 0.0001$.
CONCLUDE: We have very strong evidence that macaque neural response to monkey calls is stronger than the response to pure tones.

−1	10
−0	8
−0	110
0	011123333444
0	56667
1	0001244
1	6677
2	34
2	6

17.12. Using the mean and standard deviation found in the previous exercise, and either $t^* = 1.697$ (using df = 30 from Table C) or 1.6883 (using df = 36 from software), the 90% confidence interval is

$$\bar{x} \pm t^* \left(\frac{s}{\sqrt{37}}\right) = 45.703 \text{ to } 95.054 \text{ spikes/second} \quad (t^* = 1.697), \text{ or}$$
$$= 45.830 \text{ to } 94.927 \text{ spikes/second} \quad (t^* = 1.6883).$$

17.13. The distribution of nitrogen concentration is quite skewed, and has an outlier. With such a small sample size, we should not use t procedures.

If students find the confidence interval in spite of this, they will find it is quite wide: $225.71 \pm 122.03 = 103.7$ to 347.7 ppm. This is another indication that the t procedures are not useful here.

0	00000000000111
0	2222233
0	44
0	
0	
1	
1	
1	4

17.14. The distribution of the carbon-13 ratio is slightly skewed, but t procedures should be fairly reliable. With $\bar{x} = -2.8825$, $s \doteq 1.0360$, and df $= 23$, the 95% confidence interval is

$$\bar{x} \pm 2.069 \left(\frac{s}{\sqrt{24}}\right) = -2.8825 \pm 0.4375 = -3.320 \text{ to } -2.445.$$

−4	200
−3	9888765
−3	2
−2	8877
−2	4310
−1	85
−1	31
−0	8

17.15. (b) In real-life settings, we almost never know the true value of σ.

17.16. (c) $t = \frac{8 - 10}{4/\sqrt{20}} \doteq -2.24$

17.17. (a) The degrees of freedom are df $= n - 1 = 19$.

17.18. (a) For df = 19, 2.25 falls between the critical values 2.205 and 2.539 in Table C. This is a one-sided test, so P equals the upper tail probability, placing it between 0.01 and 0.02.

17.19. (c) Use a t distribution with $\text{df} = n - 1 = 14$.

17.20. (a) For a two-sided test with $n = 15$ and $\alpha = 0.005$, we need $\text{df} = n - 1 = 14$ and upper tail probability equal to $\alpha/2 = 0.0025$.

17.21. (a) We have df = 23 and $t^* = 2.069$, so the 95% confidence interval for μ is

$$85 \pm 2.069\left(\frac{12}{\sqrt{24}}\right) = 85 \pm 5.0680 = 79.9 \text{ to } 90.1 \text{ mg/dl}.$$

17.22. (a) The sample size should be large enough to overcome mild skewness, but an outlier will make the results unreliable.

17.23. (b) The two samples are independent; there is no matching between a male student and a female student.

17.24. (c) Robustness means that t procedures are approximately correct provided the data are an SRS.

17.25. For the student group: $t = \frac{0.08-0}{0.37/\sqrt{12}} \doteq 0.749$ (rather than 0.49). For the nonstudent group: $t = \frac{0.35-0}{0.37/\sqrt{12}} \doteq 3.277$ (rather than 3.25—this difference might be due to rounding error). From Table C, the first P-value is between 0.4 and 0.5 (software gives 0.47), and the second P-value is between 0.005 and 0.01 (software gives 0.007).

17.26. With $\bar{x} = 26.8$ and $s = 7.42$, and using either $t^* = 1.984$ (using df = 100 from Table C) or 1.9636 (using df = 653 from software), the 95% confidence interval for BMI is

$$\begin{aligned}\bar{x} \pm t^*\left(\frac{s}{\sqrt{654}}\right) &= 26.8 \pm 0.5756 = 26.2244 \text{ to } 27.3756 \quad (t^* = 1.984), \text{ or}\\ &= 26.8 \pm 0.5697 = 26.2303 \text{ to } 27.3697 \quad (t^* = 1.9636).\end{aligned}$$

17.27. **(a)** The sample size is very large, so the only potential hazard is extreme skewness. As scores range only from 0 to 500, there is a limit to how skewed the distribution could be. **(b)** From Table C, we take $t^* = 2.581$ (df = 1000), or with software, we take $t^* \doteq 2.5792$. For either value of t^*, the 99% confidence interval is $\bar{x} \pm t^*\text{SE} \doteq 240 \pm 2.84 = 237.2$ to 242.8. **(c)** Because the 99% confidence interval for μ does not include 243, we can reject H_0: $\mu = 243$ in favor of the two-sided alternative at the 1% significance level. (The evidence is a bit stronger than that: we would typically test H_0 against the one-sided alternative $\mu < 243$; for this test we find $P = 0.0032$.)

17.28. **(a)** Use df = 26:

$$114.9 \pm 2.056\left(\frac{9.3}{\sqrt{27}}\right) = 111.2 \text{ to } 118.6 \text{ mm Hg.}$$

(b) The essential assumption is that the 27 men tested can be regarded as an SRS from a population, such as all healthy white males in a stated age-group. The assumption that blood pressure in this population is Normally distributed is *not* essential because $\bar{x}$ from a sample of size 27 will be roughly Normal in any event, provided the population is not too greatly skewed and has no outliers.

17.29. **(a)** A subject's responses to the two treatments would not be independent. **(b)** We find $t = \frac{-0.326-0}{0.181/\sqrt{6}} \doteq -4.41$; with df = 5, this yields $P = 0.0069$—significant evidence of a difference.

17.30. **(a)** A stemplot shows that the data are not skewed and have no outliers. (There is a gap between 1.12 and 1.18, but as 1.18 appears three times, we would not consider it an outlier.) **(b)** Calculate $\bar{x} \doteq 1.1182$ and $s \doteq 0.04378$. We have $n = 11$, so df $= n - 1 = 10$. The critical value for a 95% confidence interval is $t^* = 2.228$, and the interval for μ is

10	77
10	88
11	11
11	22
11	
11	
11	888

$$\bar{x} \pm t^*\left(\frac{s}{\sqrt{n}}\right) = 1.1182 \pm 2.228\left(\frac{0.04378}{\sqrt{11}}\right) = 1.1182 \pm 0.0294 = 1.089 \text{ to } 1.148.$$

(c) Because this interval does not include 1, we can reject H_0: $\mu = 1$ in favor of the two-sided alternative.

17.31. **(a)** Stems can be split two ways or five ways; both stemplots are shown on the right. The second one suggests that the two highest counts might be outliers, but they are not too extreme; the use of t procedures should be fairly safe. **(b)** With $\bar{x} = 12.8\overline{3}$ and $s \doteq 4.6482$, the 90% confidence interval for μ is

0	699
1	01124
1	55
2	02

0	6
0	99
1	011
1	2
1	455
1	
1	
2	0
2	2

$$12.8333 \pm 1.796\left(\frac{4.6482}{\sqrt{12}}\right) = 12.8333 \pm 2.4099 = 10.423 \text{ to } 15.243.$$

17.32. SOLVE: The mean is $\bar{x} \doteq 25.42°$ and the standard deviation is 7.47°, and t^* is either 2.042 (using df = 30 from Table C) or 2.0262 (using df = 37 from software). The confidence interval is nearly identical in both cases:

```
TInterval
 (22.964,27.878)
 x̄=25.42105263
 Sx=7.474755999
 n=38
```

$$\bar{x} \pm t^*\left(\frac{s}{\sqrt{38}}\right) = 22.95° \text{ to } 27.89° \quad (t^* = 2.042), \text{ or}$$

$$= 22.96° \text{ to } 27.88° \quad (t^* = 2.0262).$$

CONCLUDE: We are 95% confident that the mean HAV angle among such patients is between 22.95° and 27.89°.

17.33. **(a)** The stemplot clearly shows the high outlier mentioned in the text. **(b)** Let μ be the mean difference (control minus experimental) in healing rates. We test H_0: $\mu = 0$ vs. H_a: $\mu > 0$. The alternative hypothesis says that the control limb heals faster; that is, the healing rate is greater for the control limb than for the experimental limb.

−1	3
−0	6
−0	
0	12
0	5789
1	012
1	
2	
2	
3	1

With all 12 differences, $\bar{x} = 6.41\overline{6}$ and $s \doteq 10.7065$, so $t = \frac{\bar{x}-0}{s/\sqrt{12}} \doteq 2.08$ (df = 11, $P = 0.0311$). If we omit the outlier, $\bar{x}^* = 4.\overline{18}$ and $s^* \doteq 7.7565$, so $t = \frac{\bar{x}^*-0}{s^*/\sqrt{11}} \doteq 1.79$ (df = 10, $P = 0.0520$). With all differences, the evidence is significant at the 5% level, but dropping the outlier weakens the evidence so that it is not quite significant.

Minitab output: All points

```
Test of mu = 0.00 vs mu > 0.00

Variable       N       Mean     StDev    SE Mean          T     P-Value
Diff          12       6.42     10.71       3.09       2.08       0.031
```

With outlier removed

```
Test of mu = 0.00 vs mu > 0.00

Variable       N       Mean     StDev    SE Mean          T     P-Value
Diff          11       4.18      7.76       2.34       1.79       0.052
```

17.34. **(a)** Without the outlier, the mean is $\bar{x}^* \doteq 24.76°$ while the standard deviation drops to $s^* \doteq 6.34°$. t^* is either 2.042 (using df = 30 from Table C) or 2.0281 (using df = 36 from software). The confidence interval is

```
TInterval
 (22.643,26.87)
 x̄=24.75675676
 Sx=6.339494395
 n=37
```

$$\bar{x}^* \pm t^* \left(\frac{s^*}{\sqrt{37}}\right) = 22.63° \text{ to } 26.89° \quad (t^* = 2.042), \text{ or}$$

$$= 22.64° \text{ to } 26.87° \quad (t^* = 2.0281).$$

(b) The width of the interval decreases (because the outlier raised both $\bar{x}$ and s).

17.35. **(a)** The stemplot does not show any severe evidence of non-Normality, so t procedures should be safe. **(b)** With $\bar{x} = 1.1\overline{72}$ days, $s \doteq 0.4606$ days, and $t^* = 1.812$ (df = 10), the 90% confidence interval is

$$\bar{x} \pm t^* \left(\frac{s}{\sqrt{11}}\right) \doteq 1.1727 \pm 0.2517 = 0.9211 \text{ to } 1.4244 \text{ days.}$$

0	67
0	89
1	00
1	33
1	4
1	
1	9
2	0

17.36. **(a)** The stemplot of differences shows a sharp right skew, and one or two high outliers. The t procedures should not be used. **(b)** Some students might perform the test (H_0: $\mu = 0$ vs. H_a: $\mu > 0$) despite the right skew noted in (a). If so, they should find that $\bar{x} = 156.\overline{36}$, $s \doteq 234.2952$, and $t \doteq 2.21$ (df = 10, $P = 0.0256$).

0	001223
1	0
2	1
3	
4	
5	1
6	
7	0

17.37. **(a)** We test H_0: $\mu = 0$ vs. H_a: $\mu > 0$, where μ is the mean difference (treated minus control). We use a one-sided alternative because the researchers have reason to believe that CO_2 will increase growth rate. **(b)** The mean difference is $\bar{x} = 1.916$, and the standard deviation is 1.050, so the t statistic is $t = \frac{1.916}{1.050/\sqrt{3}} \doteq 3.16$ with df $= 2$. This is significant at $\alpha = 0.05$, as the TI-83 and Minitab outputs confirm ($P = 0.044$). **(c)** For small samples, the t procedures should be used only for samples from a Normal population; we have no way to assess Normality for these data.

```
T-Test
μ>0
t=3.159644096
p=.0436279547
x̄=1.916333333
Sx=1.050493852
n=3
```

Minitab output

```
Test of mu = 0.000 vs mu > 0.000

Variable     N      Mean     StDev   SE Mean        T    P-Value
diff         3     1.916     1.050     0.607     3.16      0.044
```

17.38. **(a)** Weather conditions that change from day to day can affect spore counts. So the two measurements made on the same day form a matched pair. **(b)** Take the differences, kill room counts minus processing counts. For these differences, $\bar{x} = 1824.5$ and $s \doteq 834.1$ CFUs/m^3. For the population mean difference, the 90% confidence interval for μ is

$$1824.5 \pm 2.353\left(\frac{834.1}{\sqrt{4}}\right) = 1824.5 \pm 981.3 = 843.2 \text{ to } 2805.8 \text{ CFUs/m}^3.$$

The interval is wide because the sample is small (four days), but we are confident that mean counts in the kill room are quite a bit higher. **(c)** The data are counts, which are at best only approximately Normal, and we have only a small sample ($n = 4$).

17.39. The data contain two extreme high outliers, 5973 and 8015. These may distort the t statistic. (This exercise did not call for a stemplot, but it is shown here because it makes the outliers easily visible.)

```
0 | 1123788
1 | 00115677899
2 | 01112458
3 |
4 |
5 | 9
6 |
7 |
8 | 0
```

17.40. **(a)** The mean and standard deviation are $\bar{x} \doteq 48.25$ and $s \doteq 40.24$ thousand barrels of oil. From Table C, we take $t^* = 2.000$ (df $= 60$); using software we can obtain $t^* = 1.998$ for df $= 63$. The 95% confidence interval for μ is

$$48.25 \pm 2.000\left(\frac{40.24}{\sqrt{64}}\right) = 48.25 \pm 10.06 = 38.19 \text{ to } 58.31$$

(software value: $48.25 \pm 10.05 = 38.20$ to 58.30 thousand barrels of oil). **(b)** The stemplot confirms the skewness and outliers described in the exercise. The two intervals have similar widths, but the new interval is higher (by 2000 barrels). While t procedures are fairly robust, we should be cautious about trusting our result from (a) because of the strong skew and outliers; the computer-based method is presumably more reliable for this situation.

```
0 | 00001111111111
0 | 22222223333333333333
0 | 44444445555555
0 | 6666667
0 | 8899
1 | 01
1 |
1 | 5
1 |
1 | 9
2 | 0
```

17.41. PLAN: We will construct a 90% confidence interval for μ, the mean percent of beetle-infected seeds.

0	07
1	9
2	24689
3	666778
4	0000336
5	157
6	
7	00
8	57

SOLVE: A stemplot (right) shows a single-peaked and roughly symmetric distribution. We assume that the 28 plants can be viewed as an SRS of the population. We find that $\bar{x} \doteq 4.0786$ and $s \doteq 2.0135$. Using df = 27, the 90% confidence interval for μ is

$$4.0786 \pm 1.703\left(\frac{2.0135}{\sqrt{28}}\right) = 4.0786 \pm 0.648 = 3.43\% \text{ to } 4.73\%.$$

CONCLUDE: The beetle infects less than 5% of seeds, so it is unlikely to be effective in controlling velvetweed.

17.42. PLAN: Let μ be the mean difference (control minus experimental) in healing rates. We test H_0: $\mu = 0$ vs. H_a: $\mu > 0$. The alternative hypothesis says that the control limb heals faster; that is, the healing rate is greater for the control limb than for the experimental limb.

−1	0
−0	
−0	43
0	113334
0	7
1	02
1	
2	2
2	
3	1

SOLVE: We assume that the data can be regarded as an SRS. A stemplot of the differences shows no major deviations from Normality; the highest difference might be an outlier, but it is not too extreme, so we cautiously proceed with the t test. The mean and standard deviation of the set of differences are $\bar{x} \doteq 5.7143$ μm/hr and $s \doteq 10.5643$ μm/hr, so $t = \frac{5.7143-0}{10.5643/\sqrt{14}} \doteq 2.02$ with df = 13, for which $0.025 < P < 0.05$ (software reports 0.032). If subtraction was done in the other order, $\bar{x}$ and t are negative, but P is the same.

CONCLUDE: This is fairly strong evidence (significant at 5% but not at 1%) that altering the electric field reduces the healing rate.

Minitab output

```
Test of mu = 0.00 vs mu > 0.00

Variable      N       Mean    StDev   SE Mean        T    P-Value
HealRate     14       5.71    10.56      2.82     2.02      0.032
```

17.43. PLAN: As in the previous solution, let μ be the mean difference (control minus experimental) in healing rates. We construct a 90% confidence interval for μ.

SOLVE: With $\bar{x} \doteq 5.7143$ μm/hr, $s \doteq 10.5643$ μm/hr, and $t^* = 1.771$ (df = 13), the 90% confidence interval for μ is

$$5.7143 \pm 1.771\left(\frac{10.5643}{\sqrt{14}}\right) = 5.7143 \pm 5.0003 = 0.71 \text{ to } 10.71\ \mu\text{m/hr}.$$

CONCLUDE: We are 90% confident that the mean healing rate for control limbs exceeds the experimental limb rate by between 0.71 and 10.71 μm/hr.