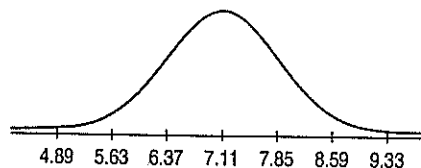


Chapter 3 Solutions

- 3.1. Sketches will vary. Use them to confirm that students understand the meaning of (a) symmetric and (b) skewed to the left.
- 3.2. (a) It is on or above the horizontal axis everywhere, and because it forms a $\frac{1}{3} \times 3$ rectangle, the area beneath the curve is 1. (b) One-third of accidents occur in the first mile: this is a $\frac{1}{3} \times 1$ rectangle, so the proportion is $\frac{1}{3}$. (c) One-tenth of accidents occur next to Sue's property: this is a $\frac{1}{3} \times 0.3$ rectangle, so the proportion is 0.1.
- 3.3. $\mu = 1.5$ —the obvious balance point of the rectangle. The median is also 1.5 because the distribution is symmetric (so that median = mean) and because half of the area lies to the left and half to the right of 1.5.
- 3.4. (a) Mean is C, median is B (the right skew pulls the mean to the right). (b) Mean B, median B (this distribution is symmetric). (c) Mean A, median B (the left skew pulls the mean to the left).

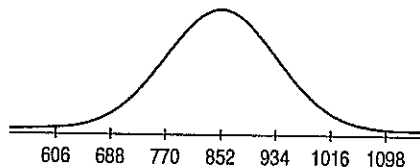
Note: For each of the three curves, the highest point(s) of the curves are marked, but are neither the mean nor the median. Those points are the modes of those distributions; your students might have heard of this other measure of center, but for most of the data considered in this text, the mode is not as useful as the mean and median.

- 3.5. Students may at first make mistakes such as drawing a half-circle instead of the correct "bell-shaped" curve, or being careless about locating the change-of-curvature (inflection) point.



- 3.6. Refer to the sketch in the solution to the previous exercise. (a) About 99.7% of male college students can run a mile in between 4.89 and 9.33 minutes—three standard deviations above and below the mean: $7.11 \pm 3(0.74) = 4.89$ to 9.33 minutes. (b) About 16% can run a mile in less than 6.37 minutes (one standard deviation below the mean); this is half of the approximately 32% of observations which do *not* fall within one standard deviation of the mean.

- 3.7. (a) In 95% of all years, monsoon rain levels are between 688 and 1016 mm—two standard deviations above and below the mean: $852 \pm 2(82) = 688$ to 1016 mm. (b) The driest 2.5% of monsoon rainfalls are less than 688 mm; this is more than 2 standard deviations below the mean.

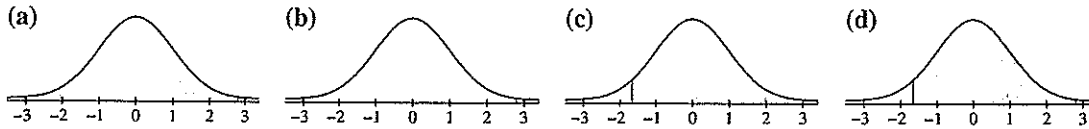


Note: This exercise did not ask for a sketch of the Normal curve, but students should be encouraged to make such sketches anyway.

- 3.8. Eleanor's standardized score is $z = \frac{680-515}{114} \doteq 1.45$, and Gerald's standardized score is $z = \frac{27-21.0}{5.1} \doteq 1.18$. Eleanor's score is higher.

3.9. First note that 6 ft is 72 inches. The z -scores are $z_w = \frac{72-64}{2.7} \doteq 2.96$ for women and $z_m = \frac{72-69.3}{2.8} \doteq 0.96$ for men. The z -scores tell us that 6 feet is quite tall for a woman, but not at all extraordinary for a man.

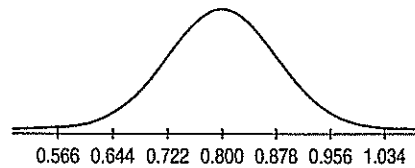
3.10. (a) 0.9978. (b) 0.0022. (c) 0.9515. (d) $0.9515 - 0.0022 = 0.9493$.



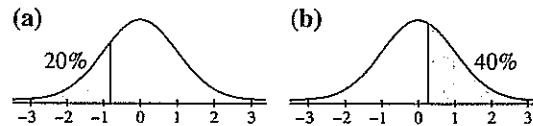
3.11. Let x be the monsoon rainfall in a given year. (a) $x \leq 697$ mm corresponds to $z \leq \frac{697-852}{82} \doteq -1.89$, for which Table A gives $0.0294 = 2.94\%$. (b) $683 < x < 1022$ corresponds to $\frac{683-852}{82} < z < \frac{1022-852}{82}$ —that is, $-2.06 < z < 2.07$. This proportion is $0.9808 - 0.0197 = 0.9611 = 96.11\%$.

3.12. Let x be the length of the thorax of a given fly.

(a) $x \geq 0.9$ mm corresponds to $z \geq \frac{0.9-0.8}{0.078} \doteq 1.28$, for which Table A gives $1 - 0.8997 = 0.1003$. (b) $0.9 < x < 1$ corresponds to $1.28 < z < \frac{1-0.9}{0.078} \doteq 2.56$. This proportion is $0.9948 - 0.8997 = 0.0951$.



3.13. (a) Search Table A for 0.20: $z \doteq -0.84$ (software gives -0.8416).
(b) Search Table A for 0.60: $z \doteq 0.25$ (software: 0.2533).



3.14. The median is the same as the mean: 0.800 mm. The quartiles are 0.67 standard deviations above and below the mean: $0.800 \pm (0.67)(0.078) \doteq 0.7477$ and 0.8523 mm.

3.15. (a) Income distributions are typically skewed to the right. In particular, there will be many countries where average income is low to moderate, but in a few countries, average income will be much higher.

3.16. (a) Mean and standard deviation tell you center and spread, which is all you need for a Normal distribution.

3.17. (b) The curve is centered at 2.

3.18. (b) Estimating a standard deviation is more difficult than estimating the mean, but among the three options, 2 is clearly too small, and 5 is clearly too large.

3.19. (b) $266 \pm 2(16) = 234$ to 298 days.

3.20. (c) 130 is two standard deviations above the mean, so 2.5% of adults have IQs of 130 or more.

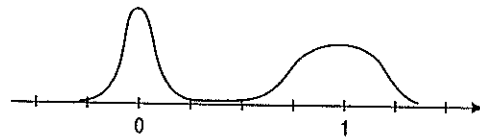
3.21. (a) Corinne's z -score is $z = \frac{118-100}{15} = 1.2$.

3.22. (c) Table A shows an area of 0.8749 for $z = 1.15$.

3.23. (b) Table A shows an area of 0.2266 below $z = -0.75$ (or 0.7734 below $z = 0.75$).

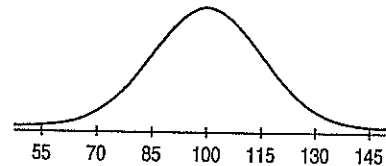
3.24. (b) Corinne's standard score is $z = \frac{118-100}{15} = 1.2$, for which Table A gives 0.8849.

3.25. Sketches will vary, but should be some variation on the one shown here: the peak at 0 should be "tall and skinny," while near 1, the curve should be "short and fat."

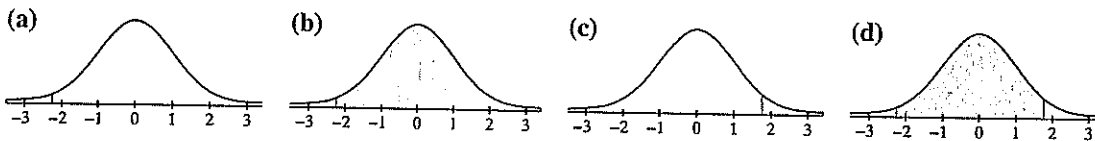


3.26. For each distribution, take the mean plus or minus two standard deviations. For mildly obese people, this is $373 \pm 2(67) = 239$ to 507 minutes. For lean people, this is $526 \pm 2(107) = 312$ to 740 minutes.

3.27. 70 is two standard deviations below the mean (that is, it has standard score $z = -2$), so about 2.5% (half of the outer 5%) of adults would have WAIS scores below 70.



3.28. (a) 0.0122. (b) $1 - 0.0122 = 0.9878$. (c) $1 - 0.9616 = 0.0384$. (d) $0.9616 - 0.0122 = 0.9494$.



3.29. (a) Search Table A for 0.80: $z \doteq 0.84$ (software: 0.8416). (b) Search Table A for 0.65: $z \doteq 0.39$ (software: 0.3853).

3.30. The proportion of students who could run a mile in 5 minutes or less is about 0.0022: $x < 5$ corresponds to $z < \frac{5-7.11}{0.74} \doteq -2.85$, for which Table A gives 0.0022.

3.31. About 0.2296: The proportion of rainy days with rainfall pH below 5.0 is about 0.2296: $x < 5.0$ corresponds to $z < \frac{5.0-5.4}{0.54} \doteq -0.74$, for which Table A gives 0.2296.

3.32. (a) Less than 2% of runners have heart rates above 130 bpm: for the $N(104, 12.5)$ distribution, $x > 130$ corresponds to $z > \frac{130-104}{12.5} = 2.08$, for which Table A gives $1 - 0.9812 = 0.0188 = 1.88\%$. (b) About 50% of nonrunners have heart rates above 130 bpm: for the $N(130, 17)$ distribution, $x > 130$ corresponds to $z > 0$.

- 3.33. About 0.9876: for the $N(0.8750, 0.0012)$ distribution, $0.8720 < x < 0.8780$ corresponds to $\frac{0.8720-0.8750}{0.0012} < z < \frac{0.8780-0.8750}{0.0012}$, or $-2.50 < z < 2.50$, for which Table A gives $0.9938 - 0.0062 = 0.9876$.
- 3.34. (a) About 93.3%: for the $N(11.5, 0.2)$ distribution, $11.2 < x < 12.2$ corresponds to $\frac{11.2-11.5}{0.2} < z < \frac{12.2-11.5}{0.2}$, or $-1.50 < z < 3.50$. Table A does not have a value for 3.50, but using the proportion given for $z = 3.49$, this proportion is $0.9998 - 0.0668 = 0.9330$.
 (b) About 98.8%: with the mean adjusted to 11.7 kg, $11.2 < x < 12.2$ corresponds to $\frac{11.2-11.7}{0.2} < z < \frac{12.2-11.7}{0.2}$, or $-2.50 < z < 2.50$, for which Table A gives $0.9938 - 0.0062 = 0.9876$.
- 3.35. About 92.92%: for the $N(18.7, 4.3)$ distribution, $x < 25$ corresponds to $z < \frac{25-18.7}{4.3} \doteq 1.47$, for which Table A gives $0.9292 = 92.92\%$.
- 3.36. At least 24.2 mpg: search Table A for the proportion closest to 0.90; this is $z \doteq 1.28$, the 90th percentile for the $N(0, 1)$ distribution. The top 10% of all vehicles are those with gas mileage at least 1.28 standard deviations above the mean: $18.7 + (1.28)(4.3) \doteq 24.2$ mpg or more.
- 3.37. The quartiles are 0.67 standard deviations above and below the mean: $18.7 \pm (0.67)(4.3) \doteq 15.8$ and 21.6 mpg.
- 3.38. The quintiles are 15.1, 17.6, 19.8, and 22.3 mpg: for the $N(0, 1)$ distribution, the quintiles are about $-0.84, -0.25, 0.25,$ and 0.84 , so we find 0.84 and 0.25 standard deviations above and below the mean; that is, $18.7 \pm (0.84)(4.3) \doteq 15.1$ and 22.3 mpg, and $18.7 \pm (0.25)(4.3) \doteq 17.6$ and 19.8 mpg.
- 3.39. Jacob's score standardizes to $z = \frac{16-21.2}{5.0} = -1.04$; this is about the 15th percentile (Table A gives 0.1492).
- 3.40. About 0.0031: a score of 1600 standardizes to $z = \frac{1600-1021}{211} \doteq 2.74$, for which Table A gives a proportion of 0.9969 (below). Therefore, the proportion above 1600 (which are reported as 1600) is about 0.0031.
- 3.41. About 2.5% of young women are taller than the mean height of young men, because 69.3 inches corresponds to a standard score (on the women's scale) of $z = \frac{69.3-64}{2.7} \doteq 1.96$, which yields 0.0250 in Table A (or round to $z = 2$ and use the 68-95-99.7 rule to get the same result).
- 3.42. About 2.9% of young men are shorter than the mean height of young women: a height of 64 inches corresponds to a standard score (on the men's scale) of $z = \frac{64-69.3}{2.8} \doteq -1.89$, which yields 0.0294 in Table A.

3.43. (a) Among men, a score of 750 corresponds to standard score $z = \frac{750-533}{116} \doteq 1.87$, so about 3.07% score 750 or more. (b) Among women, a score of 750 corresponds to standard score $z = \frac{750-499}{110} \doteq 2.28$, so about 1.13% score 750 or more.

3.44. If the distribution is Normal, it must be symmetric about its mean—and in particular, the 10th and 90th percentiles must be equal distances below and above the mean—so the mean is 250 points. If 225 points below (above) the mean is the 10th (90th) percentile, this is 1.28 standard deviations below (above) the mean, so the distribution's standard deviation is $\frac{225}{1.28} \doteq 175.8$ points.

3.45. (a) About 0.6% of healthy young adults have osteoporosis (the cumulative probability below a standard score of -2.5 is 0.0062). (b) About 31% of this population of older women has osteoporosis: the BMD level that is 2.5 standard deviations below the young adult mean would standardize to -0.5 for these older women, and the cumulative probability for this standard score is 0.3085.

3.46. (a) There are two somewhat low IQs—72 qualifies as an outlier by the $1.5 \times IQR$ rule, while 74 is on the boundary. However, for a small sample, this stemplot looks reasonably Normal. (b) We compute $\bar{x} \doteq 105.84$ and $s \doteq 14.27$ and find:

$\frac{23}{31} \doteq 74.2\%$ of the scores in the range $\bar{x} \pm 1s \doteq 91.6$ to 120.1, and

$\frac{29}{31} \doteq 93.5\%$ of the scores in the range $\bar{x} \pm 2s \doteq 77.3$ to 134.4.

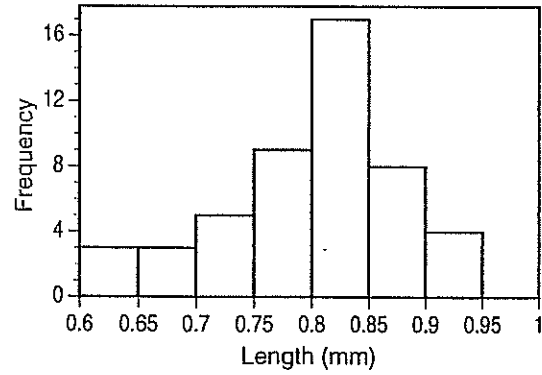
For an exactly Normal distribution, we would expect these proportions to be 68% and 95%.

7	24
7	
8	
8	69
9	13
9	68
10	023334
10	578
11	11222444
11	89
12	0
12	8
13	02

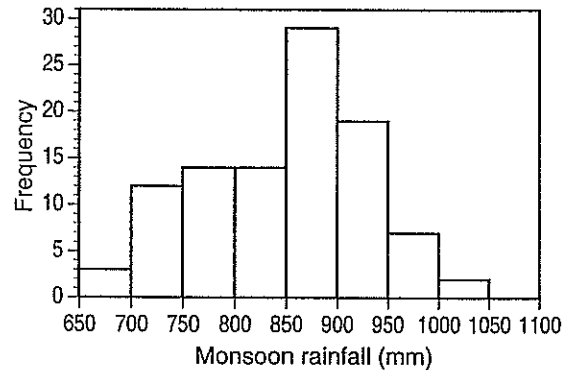
3.47. (a) The percent of scores above 27 is $\frac{149,164}{1,300,599} \doteq 11.47\%$. (b) The percent of scores greater than or equal to 27 is $\frac{149,164+50,310}{1,300,599} \doteq 15.34\%$. (c) For the $N(21.2, 5.0)$ distribution, $x > 27$ corresponds to $z > \frac{27-21.2}{5.0} = 1.16$, for which Table A gives an area of $1 - 0.8770 = 12.30\%$ above.

3.48. (a) The mean (5.43) is almost identical to the median (5.44), and the quartiles are similar distances from the median: $M - Q_1 = 0.39$ while $Q_3 - M = 0.35$. This suggests that the distribution is reasonably symmetric. (b) $x < 5.05$ corresponds to $z < \frac{5.05-5.43}{0.54} \doteq -0.70$, and $x < 5.79$ corresponds to $z < \frac{5.79-5.43}{0.54} \doteq 0.67$. Table A gives these proportions as 0.2420 and 0.7486. These are quite close to 0.25 and 0.75, which is what we would expect for the quartiles, so they are consistent with the idea that the distribution is close to Normal.

3.49. (a) One possible histogram is shown on the right. (b) The mean is $\bar{x} \doteq 0.8004$, the median is $M = 0.8$, the standard deviation is $s \doteq 0.0782$, and the quartiles are $Q_1 = 0.76$ and $Q_3 = 0.86$ (all in millimeters). \bar{x} and M are quite close, and the distances from the median to each quartile are similar: $M - Q_1 = 0.04$ and $Q_3 - M = 0.06$. This is what we expect for a symmetric distribution. (c) $0.76 < x < 0.86$ corresponds to $\frac{0.76-0.8004}{0.0782} < z < \frac{0.86-0.8004}{0.0782}$, or $-0.52 < z < 0.76$, for which Table A gives $0.7764 - 0.3015 = 0.4749$. Of the 49 measurements, the proportion falling between 0.76 and 0.86 (inclusive) is $\frac{26}{49} \doteq 0.5306$. (This includes 8 observations equal to 0.76.)



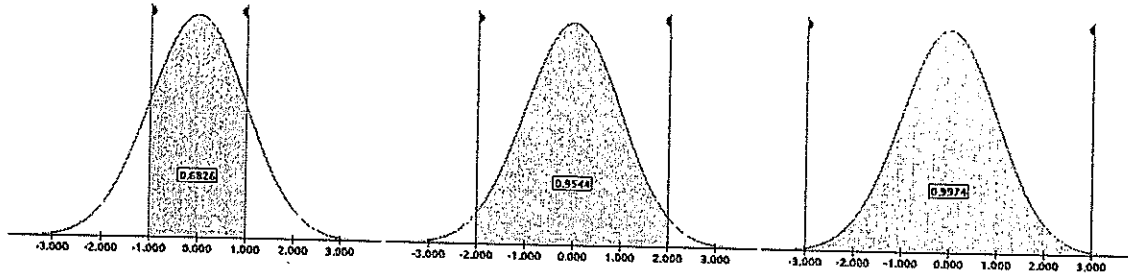
3.50. (a) One possible histogram is on the right. The mean is $\bar{x} \doteq 847.58$ mm, and the median is $M = 860.8$ mm. (b) The histogram shows a left skew; this makes the mean lower than the median.



3.51. See also the solution to Exercise 2.47. Both stemplots suggest that the distributions may be slightly skewed to the right, rather than perfectly symmetric (as a Normal distribution would be).

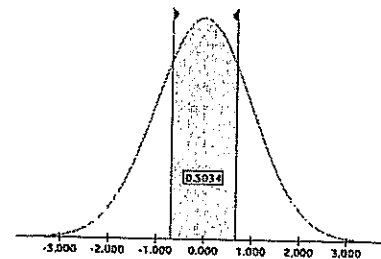
Loose	Intermediate
2	2 99
3	3 0111111
3	3 2333
3	3 4445
3	3 6
3 9999	3 8
4 0011111	4
4 22233	4 2
4 44	
4	
4	
4 89	

- 3.52. (a) The applet shows an area of 0.6826 between -1.000 and 1.000 , while the 68-95-99.7 rule rounds this to 0.68. (b) Between -2.000 and 2.000 , the applet reports 0.9544 (compared with the rounded 0.95 from the 68-95-99.7 rule). Between -3.000 and 3.000 , the applet reports 0.9974 (compared with the rounded 0.997).



- 3.53. Because the quartiles of any distribution have 50% of observations between them, we seek to place the flags so that the reported area is 0.5. The closest the applet gets is an area of 0.5034, between -0.680 and 0.680 . Thus the quartiles of any Normal distribution are about 0.68 standard deviations above and below the mean.

Note: Table A places the quartiles at about ± 0.67 ; other statistical software gives ± 0.6745 .



- 3.54. Placing the flags so that the area between them is as close as possible to 0.80, we find that the A/B cutoff is about 1.28 standard deviations above the mean, and the B/C cutoff is about 1.28 standard deviations below the mean.

