Appendix D: Binomial Theorem

Pascal's Triangle

Factorial: $\boxed{n! = n(n-1)! \quad n \in \mathbb{N} \quad , \; 0! := 1 \\ = n(n-1)(n-2)\cdots 3\cdot 2\cdot 1}$

ex $\quad 5! = 5\cdot 4\cdot 3\cdot 2\cdot 1$

$$\boxed{\binom{n}{r} = \frac{n!}{r!\,(n-r)!}}$$

- Pascal's Δ is full of $\binom{n}{r}$ #s
  n = row # , r = col # for that row

Binomial Thm

$$(a+b)^n = \binom{n}{0}a^n + \binom{n}{1}a^{n-1}b + \binom{n}{2}a^{n-2}b^2 + \cdots + \binom{n}{n-1}ab^{n-1} + \binom{n}{n}b^n$$

for any $n \in \mathbb{Z}^+$

MWSD

Appendix D (cont)

Ex 1 Evaluate.

(a) $\binom{8}{5}$

(b) $\dfrac{52!}{4!\,(52-4)!}$

(c) $\dfrac{10!}{3!\,7!}$

(d) # of outcomes for 5-coin toss w/ 2 heads & 3 tails.

Ex 2 Expand (multiply out).

$(x-3)^6$

Appendix D: (cont)

Ex 3 Expand

(a) $(xy^{-1} - 2y^{-2})^8$

(b) $(\sqrt{2}\, x^2 + y^{3/5})^6$

Partial Fraction Decomposition

ex (forward)

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=?$$

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{4}=\frac{6}{12}+\frac{4}{12}+\frac{3}{12}=\frac{13}{12}=\frac{13}{2^2\cdot 3}$$

(backward)

$$\frac{13}{2^2\cdot 3}=\frac{A}{2}+\frac{B}{4}+\frac{C}{3}$$

then try to solve for A, B & C

$$13=A(6)+B(3)+C(4)$$

We'll do this instead for rational expressions.

Ex1 $\dfrac{4x-1}{x(x-1)}$ decompose this into separate fractions

$$\frac{4x-1}{x(x-1)}=\frac{A}{x}+\frac{B}{x-1}$$

- For linear factors (in denominator), numerator is constant (even for repeated linear factors)
- For quadratic factors (in denominator) numerator is a linear polynomial.

PFD (cont)

EX2 Do PFD for these.

(a) $$\frac{3x^3+13x^2+6x+12}{x(x+3)(x^2+1)}$$

(b) $$\frac{2x^3+12x+24}{(x+1)^2(x^2+9)}$$

PFD (cont)

Ex 3 Set these up for PFD.

(a) $$\frac{1}{x^3(x+5)^2(x^2+7)^2}$$

(b) $$\frac{-5x+3}{(x-1)^2(2x^2-5x-3)}$$