# 2.3 Graph of a Function

## Vocab/Defns

① Graph of a function $f(x)$ $\Rightarrow$ all pts $(x, y) \ni y = f(x)$ for all $x$ in domain of $f$.

② y-intercept $\Rightarrow$ the pt $(0, b)$ on $y = f(x)$

③ x-intercept $\Rightarrow$ the pt $(a, 0)$ on $y = f(x)$
a is also called zero/root of the function

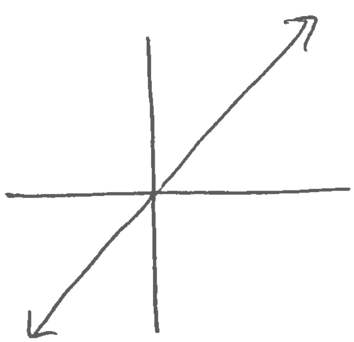
④ $f$ is increasing if $f(x_1) < f(x_2)$ whenever $x_1 < x_2$

$f$ is decreasing if $f(x_1) > f(x_2)$ whenever $x_1 < x_2$

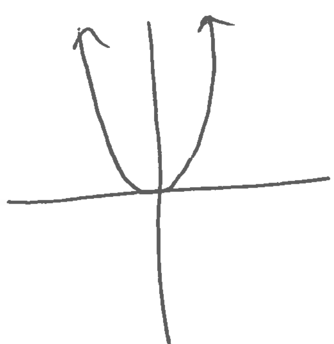
To find:

| domain | range |
|---|---|
| • solve for y | • solve for x |
| • look for restrictions on x | • look for exclusions of y |

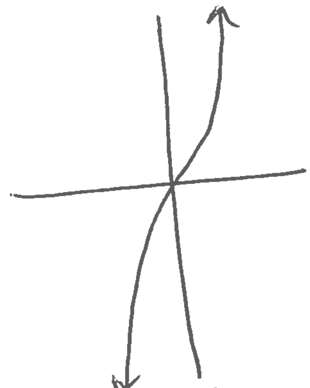
## Base Graphs



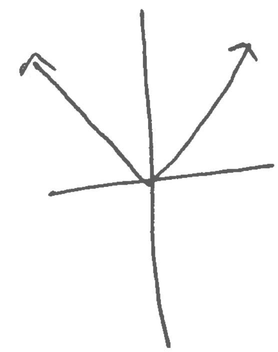
$y = x$



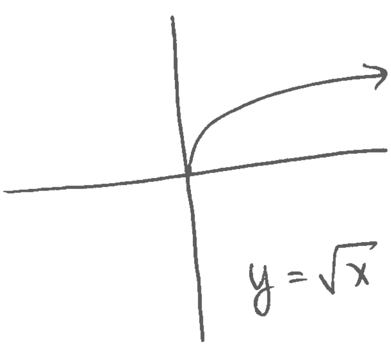
$y = x^2$



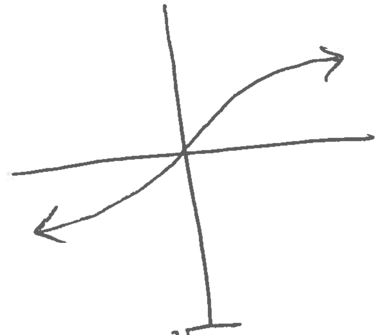
$y = x^3$



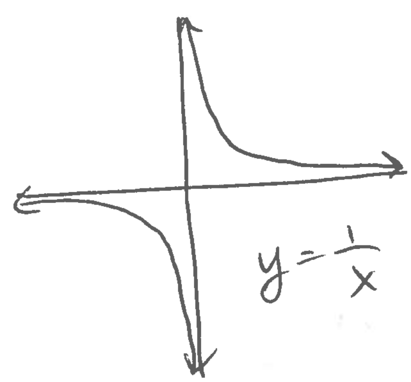
$y = |x|$



$y = \sqrt{x}$



$y = \sqrt[3]{x}$



$y = \frac{1}{x}$

## 2.3 (cont)

### More Vocab/Defns

(5) even fn: $f(-x)=f(x)$ (has symmetry wrt y-axis)

(6) odd fn: $f(-x)=-f(x)$ ( " " " origin)

(7) avg rate of change on $[a,b]$: $\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}$

(8) turning pt: where a graph "turns" from increasing to decreasing or vice versa

Ex 1 Are the fns equal? $f(x) = \frac{(3x+1)(x-4)}{x-4}$ and $g(x) = 3x+1$

Ex 2 Classify each of the base graphs (pg 7) as even, odd or neither.

2.3 (cont)

Ex 3 Find domain.

(a) $f(x) = \frac{2x+1}{(x-2)^2}$

(b) $g(x) = \frac{3x^2+1}{5x^2+2}$

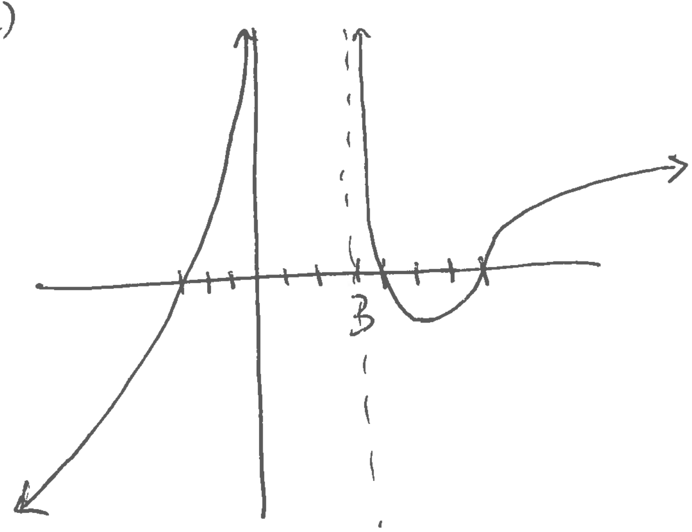
(c) $h(x) = \sqrt{7-2x}$

(d) $k(x) = \sqrt{2+x-x^2}$

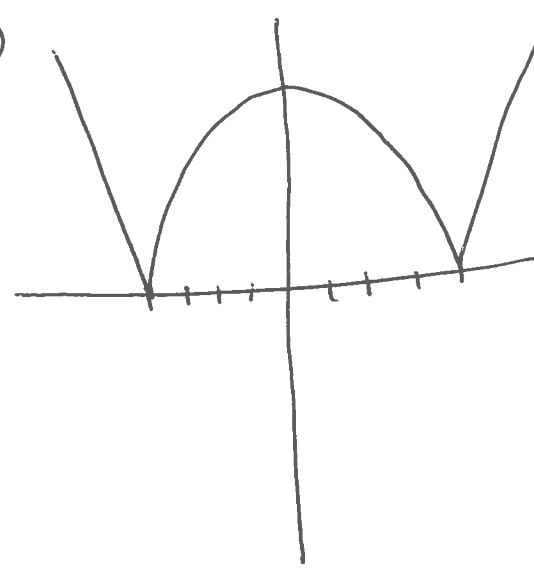
## 2.3 (cont)

**Ex 4** State domain, intercepts, turning pts, intervals where graph is increasing and decreasing.

(a)




(b)



**Ex 5** Find avg rate of change from 3 to 3+h, for $f(x) = x^3$

# 3.6 Transformations of Graphs

## TYPES OF TRANSFORMATIONS TO $y = f(x)$

(Assume $c$ is a constant such that $c \in \mathbb{R}$, $c > 0$.)

(For all examples in the last column of this table, we'll use $y = f(x) = x^2$ as the base or parent function graph.)

| | | |
|---|---|---|
| **1. Shift:** $h(x) = f(x) \pm c$ | Shifts graph up or down by $c$ units (if we add $c$, shift up; if we subtract $c$, shift down) | $y = x^2 + 2$ shifts graph 2 units up |
| $h(x) = f(x \pm c)$ | Shifts graph left or right by $c$ units (if we add $c$, shift left; if we subtract $c$, shift right) | $y = (x - 3)^2$ shifts graph 3 units right |
| **2. Reflection:** $g(x) = -f(x)$ | Reflects graph vertically (across $x$-axis) | $y = -x^2$ reflects graph vertically |
| $g(x) = f(-x)$ | Reflects graph horizontally (across $y$-axis) | $y = (-x)^2$ reflects graph horizontally |
| **3. Stretch/Shrink:** $k(x) = cf(x)$ | Stretches/shrinks graph vertically (if $c > 1$, it's a stretch; if $0 < c < 1$, it's a shrink) | $y = 5x^2$ stretches graph vertically by factor of 5 |
| $k(x) = f(cx)$ | Stretches/shrinks graph horizontally (if $c > 1$, it's a shrink; if $0 < c < 1$, it's a stretch) | $y = (4x)^2$ shrinks graph horizontally by one quarter |

***Note:*** ALL the vertical effects or changes to the graph appear "outside" the function, that is, outside the base or parent function that defines the overall shape. ALL the horizontal effects or changes to the graph appear "inside" the function, that is, before we perform the essence of the function. In the above examples, the main shape of the graph is the parabola given by $y = f(x) = x^2$. So, any algebraic change that happens before we square anything is "inside" the function, and any change that happens after the square is "outside" the function.

Note also that all the vertical shifts, stretches and shrinks are "intuitive," meaning that they're as expected. Adding two outside the function, for example, shifts the graph up, and we would expect a positive vertical shift to be up. Also, the horizontal shifts, stretches and shrinks are all "counter-intuitive," meaning that they're the opposite of what we'd expect. For example, adding three inside the function shifts the graph to the left by three units, which is perhaps the opposite of what one would expect from a positive horizontal change. Adding three inside the function means a smaller $x$-value is needed to produce the same $y$-value as before the shift. Because a smaller $x$-value is needed, we shift left instead of right.

Warning: Make sure to do all the reflections and shrink/stretch transformations FIRST before the shifts, when graphing these transformed functions. If you do the shifts first, you can get the wrong graph very easily.

Keep in mind that we can always use the default, back-up strategy of plotting lots of points and connecting the dots to graph any function. This method of understanding the transformations of graphs, along

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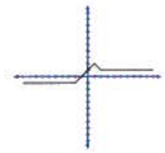
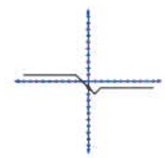
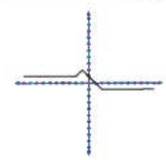
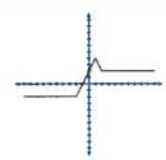
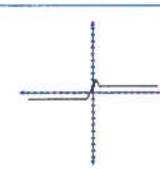
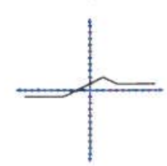
| TRANSFORMATION | EFFECT | EXAMPLE |
|---|---|---|
| $f(x)$ | Base graph | $f(x)$  |
| $h(x) = f(x) + c$ | Shifts graph *up* by $c$ units<br>(down if $c$ is negative) | $h(x) = f(x) + 2$  |
| $h(x) = f(x + c)$ | Shifts graph *left* by $c$ units<br>(right if $c$ is negative) | $h(x) = f(x - 2)$  |
| $h(x) = -f(x)$ | Reflects graph vertically | $h(x) = -f(x)$  |
| $h(x) = f(-x)$ | Reflects graph horizontally | $h(x) = f(-x)$  |
| $h(x) = c \cdot f(x)$ | Stretches graph vertically by a factor of $c$<br>(shrinks if $0 < c < 1$) | $h(x) = 2f(x)$ <br>$h(x) = \frac{1}{2}f(x)$  |
| $h(x) = f(c \cdot x)$ | Shrinks graph horizontally by a factor of $c$<br>(stretches if $0 < c < 1$) | $h(x) = f(2x)$ <br>$h(x) = f\left(\frac{1}{2}(x)\right)$  |

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2.3 (cont)

Ex 6 (optnl) If $f$ is increasing throughout its domain, prove that $f$ is one-to-one.

Pf Assume $x_1$ and $x_2$ are both in domain of $f(x)$ such that $x_1 \neq x_2$.

Then either $x_1 > x_2$ or $x_1 < x_2$.

Case 1: $x_1 < x_2$

By defn of increasing, $f(x_1) < f(x_2)$.

$\Rightarrow$ $f(x_1) \neq f(x_2)$

$\Rightarrow$ by defn, $f(x)$ is 1-1.

Case 2: $x_1 > x_2$

Then by defn of increasing $f(x_2) < f(x_1)$

$\Rightarrow$ $f(x_1) \neq f(x_2)$

$\Rightarrow$ again by defn, $f(x)$ is 1-1. //

# 2.4 Transformations of Functions

## Vocab / Defns

① Translation / Shift: $y-k = f(x-h)$ $\qquad k>0,\ h>0$

$y-k$: shift up $k$

$x-h$: shift $h$ to right

② Reflection: (a) $y = -f(x)$ vertical reflection (across $x$-axis)

(b) $y = f(-x)$ horizontal reflection (across $y$-axis)

③ Stretch/Shrink (or Dilation/Compression):

(a) $y = a f(x)$ vertical stretch if $a>1$

" shrink if $0<a<1$

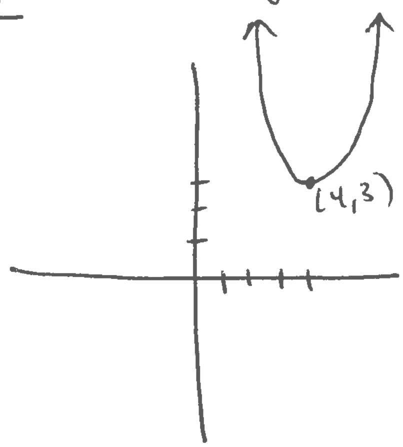
(b) $y = f(ax)$ horizontal stretch if $0<a<1$

" shrink if $a>1$

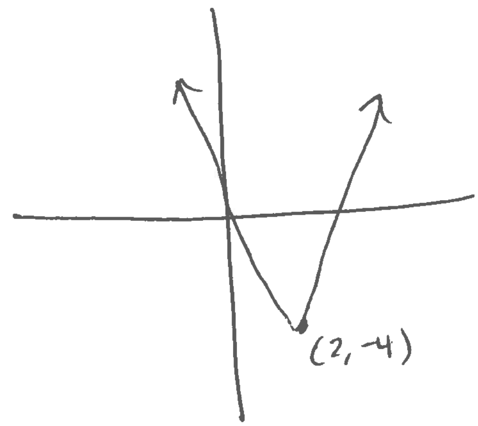
## 2.4 (cont)

**Ex 1** write eqn of $f(x)$ based on graph.

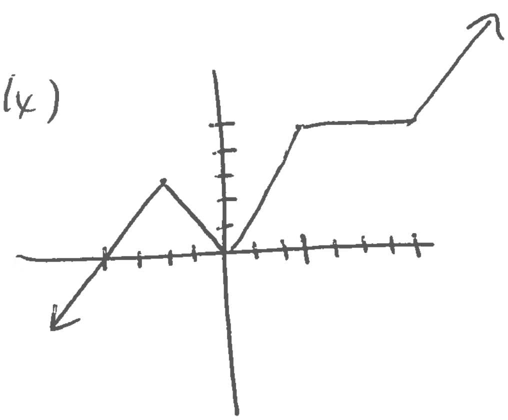
(a)




(b)




**Ex 2** Given $f(x)$



Graph

(a) $y-1=f(x-2)$

(b) $y=\frac{1}{2}f(-x)+3$

2,4 (cont)

Ex 3 Graph these curves.

(a) $y = -3|x+4|$

(b) $y = \frac{-1}{2}(x+2)^2 + 3$

(c) $y = -\sqrt{x+4}$