2.2 Intro to Functions

Vocab / Notation / Defns

function: a rule that assigns a single element from the **range** to a each element in the **domain**.
$y = f(x)$ "f of x"

domain: set of allowable inputs

range: set of outputs

Remark: ① a function can also be considered a mapping from the domain to the range.
② a function can be written as a set of ordered pairs (points) (x, y) such that no two ordered pairs have the same x-value.

independent variable: the input of a function

dependent variable: the output of a function; its value depends on the value of the input

Remark: In $y = f(x)$ notation, it's clear that x is the input (indep. var.) and y is the output (dep. var.).

one-to-one function: If a function f maps X to Y $(f: X \to Y)$ ∋ for any distinct elements x_1 & x_2 in X $f(x_1) \neq f(x_2)$, then f is one-to-one.

(∋ ≡ such that)

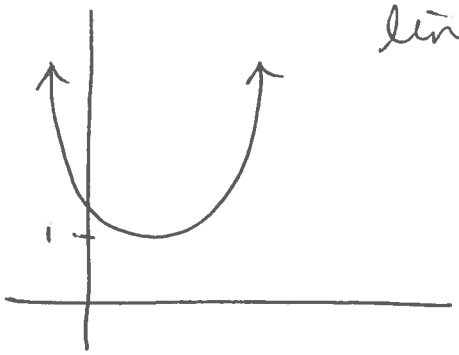
1-1 ≡ one-to-one ; fn ≡ function

2.2 (cont)

graphically, a fn passes the vertical line test and a 1-1 fn passes the horizontal line test as well

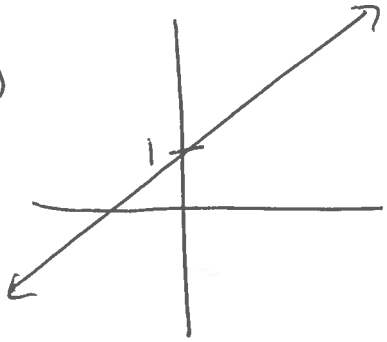
Ex 1

(a)



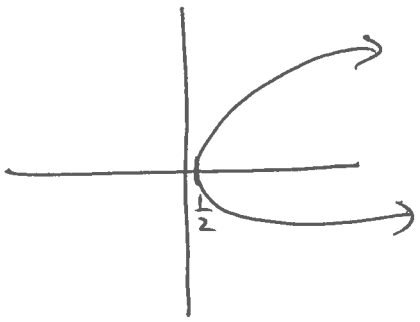
Is this a fn? Is it 1-1?
Domain? Range?

(b)



Is this a fn? Is it 1-1?
Domain? Range?

(c)



Is this a fn? Is it 1-1?
Domain? Range?

(d) $X = \{1, 2, 3, 4, 5\}$ $Y = \{-1, 0, 2, 3\}$

$f(1) = -1$ $f(4) = 2$
$f(2) = 0$ $f(5) = 3$
$f(3) = -1$

Is $f: X \to Y$ a fn? Is it 1-1?

2.2 (cont)

Ex 2 Find mathematical fn, and state domain.

"For each number x in the domain, the corresponding range value y is found by adding one to the domain value and then dividing that result into five added to five times the domain value."

Typical restrictions to domain:

Typical Types of Fns

① **polynomial**: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ (of degree n)

$n=1$ $f(x) = ax+b$ (linear) $a \neq 0$

$n=2$ $f(x) = ax^2+bx+c$ (quadratic) $a \neq 0$

$n=3$ $f(x) = ax^3+bx^2+cx+d$ (cubic) $a \neq 0$

② **rational**: $f(x) = \frac{p(x)}{d(x)}$, $d(x) \neq 0$

where $p(x)$ & $d(x)$ are polynomials

③ **power fn**: $f(x) = x^r$

integral powers, $r = n \in \mathbb{Z}^+$

reciprocal powers, $r = -n$, $n \in \mathbb{Z}^+$

roots $r = \frac{m}{n}$ $m, n \in \mathbb{Z}^+$, $n \neq 0$ or $f(x) = \sqrt[n]{x^m}$ $n \neq 0$

2.2 (cont)

more types of fns

④ Exponential: $f(x)=b^x$, $b>0$

⑤ Logarithmic: $f(x)=\log_b x$, $b>0$

⑥ Trigonometric: ex $f(x)=\sin x$

Ex 3 let $f(x)=3x+2$, $g(x)=2x^2-1$

find

(a) $f(1)$

(b) $g(\sqrt{3})$

(c) $f(t^2-3t)-g(t+2)$

(d) $\dfrac{f(x+h)-f(x)}{h}$

Difference Quotient

For $f(x)$, the difference quotient is

$$\frac{f(x+h)-f(x)}{h}$$

Geometric Interpretation:

2.2 (cont)

Ex 4 True or false? Explain.

(a) $f(x-1) = f(x) - 1$

(b) $f(x+h) = f(x) + h$

(c) $\dfrac{f(x+h)}{f(x)} = h$

(d) $f(3x) = 3f(x)$

2.2 (cont)

Ex 5 Let $R(x) = 3x^2 + 3x^{-2} - x - x^{-1}$. Show that $R(\frac{1}{x}) = R(x)$.