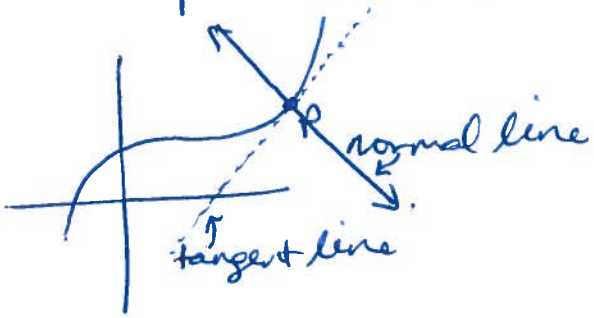
3.1 Linear Functions

Vocab/Defn

linear fn: a fn f is linear if $f(x) = mx + b$ where $m, b \in \mathbb{R}$.

(note: ① if $m = 0$, then we have $f(x) = b$ which is a constant fn, that graphs into horizontal line.
② a vertical line is not a fn (it fails VLT) and is not included in defn of linear fn.)

normal line: a line $\perp$ to tangent & curve at particular pt P. ($\perp$ = perpendicular)



a **tangent line** touches the curve at a single pt (locally)

secant line goes through curve at two pts

Ex 1 Find linear fn that satisfies $f(-2) = 6$ and $f(3) = 12$.

3.1 (cont)

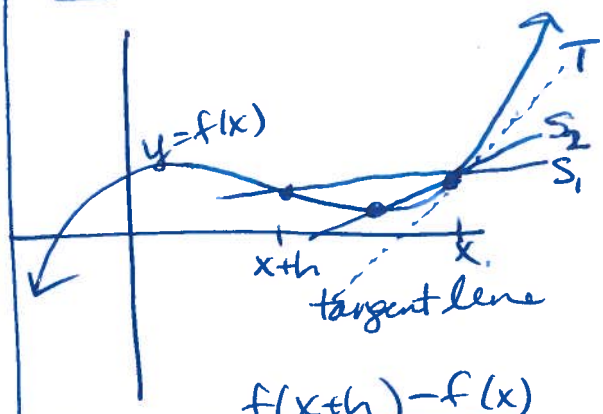
- slope
- rate
- speed
- velocity
- rate of change

are all basically synonyms

Ex 2 Find velocity of line through (2,136) and (5,340)

Ex 3 Find slope of tangent line to curve $f(x) = \frac{-1}{x}$ at $x = \frac{1}{2}$.

tangent line slope



$$m_{secant} = \frac{f(x+h)-f(x)}{h}$$

$$m_{tangent} = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h}$$

3.1 (cont)

Ex 4 Find eqn of tangent line to curve $f(x) = -x^2$ when $x = 1$.

Ex 5 Find eqn of normal line to curve $f(x) = -x^2$ at $x = 1$.

3.2 Quadratic Functions

Vocab / Defn

quadratic fn: a fn f is quadratic if

$$f(x) = ax^2+bx+c, \quad a,b,c \in \mathbb{R}, \quad a \neq 0.$$

max value: If $y = ax^2+bx+c$, $a<0$, then max value of y is located at vertex.

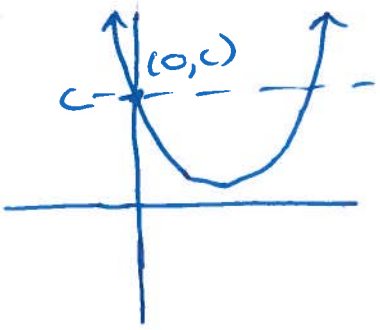
min value: If $y = ax^2+bx+c$, $a>0$, then min value of y is located at vertex.

general form of parabola: $y = a(x-h)^2+k$

(h,k) = vertex

For $f(x) = y = ax^2+bx+c$, if $x=0$, we get $y=c$

By symmetry, there is one other pt where $y=c$.



$\Rightarrow \quad c = ax^2+bx+c$

$0 = ax^2+bx$

$0 = x(ax+b)$

$\Rightarrow x=0$ or $ax+b=0$

$ax = -b$

$x = \frac{-b}{a}$

So we have 2 pts on graph $(0,c)$ and $\left(\frac{-b}{a}, c\right)$

and by symmetry, vertex must be halfway between those pts

3.2 (cont)

$\Rightarrow$ vertex at $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

* also see explanation in book on pg 171 because its slightly different argument

Ex 1 Graph these parabolas, state max/min value.

(a) $y = 5x^2$

(b) $y = \frac{1}{5}x^2$

(c) $y = \frac{1}{3}(x+1)^2 + 2$

(d) $y = 2x^2 - 4x + 5$

3.2 (cont)

Ex 2 Graph & find vertex.

(a) $y = -3x^2 - 30x - 76$

(b) $2x^2 - 4x + 3y + 11 = 0$

Ex 3 The sum of two numbers is 17. Express the product of these numbers as a fn of a single variable.

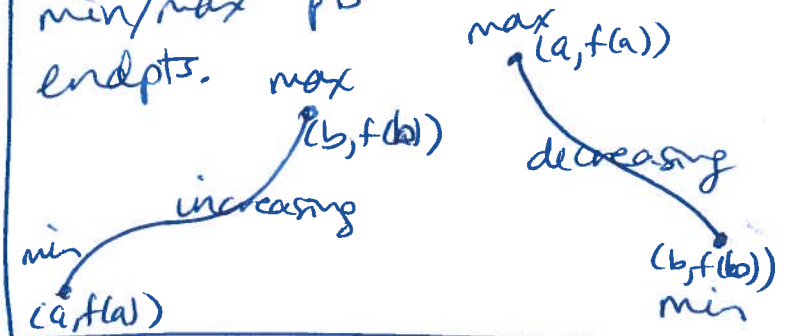
3.3 Optimization Problems

Ex 1 The difference of 2 numbers is 8. Find smallest possible product.

distance between 2 pts (x_1, y_1) and (x_2, y_2) is minimized or maximized when

$d^2 = (x_1 - x_2)^2 + (y_1 - y_2)^2$ is minimized or maximized.

If $f(x)$ is continuous and increasing (or decreasing) on $[a, b]$, then min/max pts occur at endpts.



Ex 2 A profit fn P is $P(x) = 100x - 2x^2 - 600$.
Find max profit.

Ex 3 A truck is 250 miles due east of a sports car and is traveling west at 60 mi/hr (constant). Meanwhile the sports car is heading north at 80 mi/hr. When will truck + car be closest to each other? What is min. distance between them?

3.3 (cont)

Ex 4 According to postal regulations, the girth plus the length of a parcel sent by 4th-class mail may not exceed 108 in. What is the largest possible volume of a rectangular parcel box w/ two square sides that can be sent?