

## 4.1 Rational Fns

Vocab rational fn  $f$  is quotient of two polynomials  
 $f(x) = \frac{P(x)}{D(x)}$ ,  $D(x) \neq 0$  ( $P(x)$  &  $D(x)$  are polynomials)

### Asymptotes

VA (1) vertical: of form  $x = a$  where  $D(a) = 0$

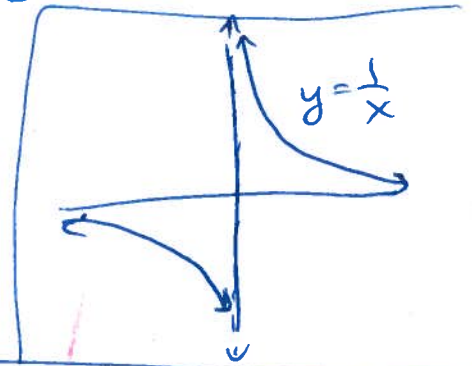
for  $f(x) = \frac{P(x)}{D(x)}$  and there are no common factors between  $P(x)$  &  $D(x)$ .

HA (2) horizontal: of form  $y = L$  where  
 $\lim_{x \rightarrow \pm\infty} f(x) = L$ , i.e. the horizontal line that is approached as  $x$  gets huge (in either direction)

OA (3) oblique: (sometimes called skew or slant asymptote)

of form  $y = mx + b$  such that

$$f(x) = \frac{P(x)}{D(x)} = mx + b + \frac{r}{D(x)}$$



Remarks (1) graph can never touch or cross

VA, no exceptions.

(2) graph can touch or cross HA/OA but it won't as  $x$  gets "huge enough"

## 4.1 (cont)

Ex 1 Graph.

(a)  $y = \frac{1}{x}$  and  $y = \frac{-4}{x}$  on same axes

$$(b) y = \frac{(x-3)(x+1)(x+5)}{(x+1)(x+5)}$$

## 4.1 (cont)

Ex 2 Graph. Find VA/AA/OA, if any.

(a).  $y = \frac{x^2 - x - 12}{x - 4}$

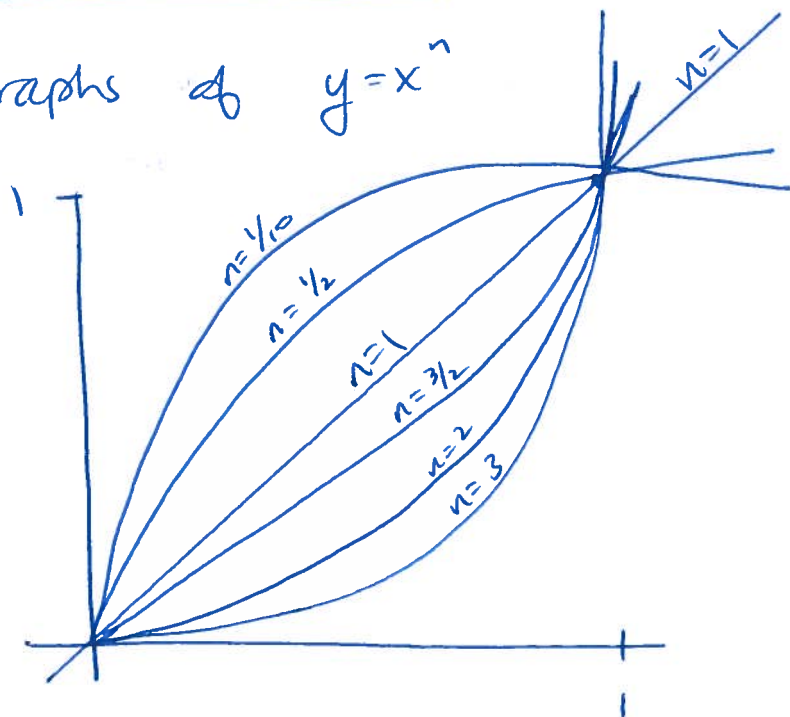
(b)  $y = \frac{(x+3)(x-1)(x+2)}{(x-1)(x-2)}$

## 4.1 (cont)

Ex 3 Consider a line w/ slope  $m$  passing through the parabola  $y=x^2$  in  $\mathbb{Q}^1$  and pt  $(0, -2)$ . Find  $m$  so that  $m$  is minimum.

## 4.2 Radical Fns

Graphs of  $y = x^n$



Note: They all go through (1, 1).

Ex 1 Graph.

(a)  $y = x^{2/3}$  and  $y = -(x-2)^{2/3}$  on same axes

4.2 (cont)

Ex 1 (cont)

$$(b) y = \sqrt[3]{x+3} - 1$$

Ex 2 Graph  $y = \frac{1}{\sqrt{1-x^2}}$

4.2 (cont)

Ex 3

Graph

$$y = \sqrt{(x-3)(x-6)(x-9)}$$

## 4.3 Real Roots of Rational & Radical Eqs

### Strategy (Rational Eqs)

- ① State domain.
- ② Multiply both sides of eqn by LCD to get rid of fractions.
- ③ Solve eqn leftover.
- ④ Check solns against domain & throw out any disallowed values

### Strategy (Radical Eqs)

- ① state domain
- ② Isolate radical expression on one side of eqn.
- ③ Raise both sides to a power that "undoes" radical.
- ④ Solve eqn leftover.
- ⑤ Check solns against domain & throw out any disallowed values
- ⑥ Plug in answers to see if they work.  
(for even roots)

Ex 1 Solve  $\frac{16}{x+5} + \frac{4}{5-x} = \frac{5-3x}{x^2-25}$



### 4.3 (cont)

Ex 2 Solve  $-x + \sqrt{x+8} = 2$

Ex 3  $\frac{x-3}{x-2} + \frac{x-1}{x} = \frac{22x-110}{3x^2-15x}$

4.3 (cont)

Ex 4 Solve

$$\sqrt{x-2} = \sqrt[4]{x^2-6x+1}$$

Ex 5  $x^{1/2} + 6x^{-1/2} + 5 = 0$

4.3 (cont)

Ex 6

$$\sqrt[3]{3x+1} - 1 = 0$$

Solve.