4.4 Exponential Fns

Defn $f(x)=b^x$ is an exponential fn w/ base b. domain: $x \in \mathbb{R}$

WARNING — Do not confuse an exponential fn w/ a power fn.

ex exponential: $y = 5^x$ ← variable in exponent

power: $y = x^5$ ← variable in base w/ fixed exponent

Rules of Exponents

① $b^p b^q = b^{p+q}$

② $\frac{b^p}{b^q} = b^{p-q}$

③ $(b^q)^p = b^{qp}$

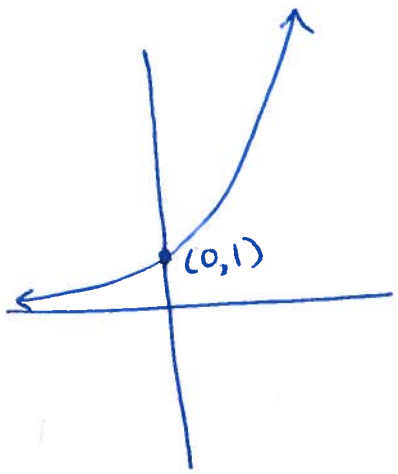
④ $(ab)^p = a^p b^p$

⑤ $\left(\frac{a}{b}\right)^p = \frac{a^p}{b^p}$

Ex 1 Graph on same axes $y = 2^x$ and $y = \left(\frac{1}{2}\right)^x$

General shape of exponential fn.

$y = b^x$, $b > 1$



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4.4 (cont)

Ex 2 Graph $y = 3^{x-1} + 4$

Compound Interest

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

A = future value of acct.
P = present value (principal)
r = annual interest rate
n = # compoundings / yr
t = # yrs

Continuous Compounding

$$A = Pe^{rt}$$

Interesting Fact

$$\lim_{n \to \infty} \left(1 + \frac{1}{n}\right)^n = e$$

n	$\left(1 + \frac{1}{n}\right)^1$
1	2
2	2.25
4	2.44
12	2.61
360	2.71

4.4 (cont)

Ex 3 If I invest $5,000 for 18 years earning 6.3% interest compounded quarterly, how much is it worth at the end?

4.5 Logarithmic Fns

Defn $x = \log_b A \iff b^x = A$, $b>0, b\neq 1, A>0$

read "log base b of A"

① $b^x = b^y \iff x = y$

② $\log_a x = \dfrac{\log_b x}{\log_b a}$

③ $\log x$ means $\log_{10} x$

④ $\ln x$ means $\log_e x$

⑤ $\log_b b^x = x$

⑥ $b^{\log_b x} = x$

} log and exponential of same base are inverse fns

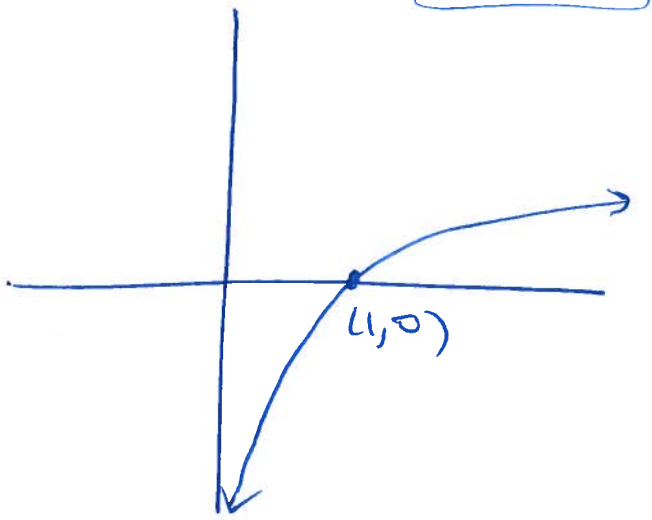
Ex 1 Rewrite in logarithmic form

(a) $\frac{1}{3} = 9^{-1/2}$

(b) $5^4 = 625$

General shape of $y = \log_b x$

domain: $x>0$



4.5 (cont)

Ex 2 Rewrite in exponential form

(a) $\log_2 32 = 5$

(b) $\log_3\left(\frac{1}{81}\right) = -4$

Ex 3 Evaluate.

(a) $\log_\pi \sqrt[3]{\pi}$

(b) $\log_{217} 1$

Ex 4 Rewrite in base 10.

(a) $\log_5 31$

(b) $\log_3 2.7$

4.5 (cont)

Ex 5 Graph (a) $y = \log_5(x+2) - 3$

(b) $y = |\log x|$

4.6 Logarithmic Eqns

Remember Defn of log: $x^a = b \Leftrightarrow \log_x b = a$

Logarithmic Properties

$A, B, b > 0$, $b \neq 1$, $p \in \mathbb{R}$

① $\log_b(AB) = \log_b A + \log_b B$

② $\log_b\left(\frac{A}{B}\right) = \log_b A - \log_b B$

③ $\log_b(A^p) = p \log_b A$

WARNING $\log_b(A+B) \neq \log_b(A) + \log_b(B)$

and $(\log_b A)^p \neq p \log_b A$

Ex 1 Solve $\log_5(x-1) = 2$

Strategy to Solve Log Eqns

① Rewrite all log expressns as one log statement

② (i) Rewrite as exponential form using Defn of log

or (ii) exponentiate both sides (w/ same base as log)

③ finish solving

④ check answer!!!

★ $\log_b \heartsuit = \log_b ★$

$\Leftrightarrow \quad \heartsuit = ★$

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4.6 (cont)

Ex 2 Use log properties to contract.

(a) $\frac{1}{2}\log(3x+1) - [\log(x+1) - \frac{3}{2}\log(3x+1)]$

(b) $\log(x^3-8) - \log(x^2+2x+4)$

Ex 3 Solve $5\ln x - 6 = 104$

4.6 (cont)

Ex 4 Solve.

(a) $2 \ln x^2 = 4$

(b) $\log(\log x) = 1$

(c) $\log_7 x - \frac{1}{2}\log_7 4 = \frac{1}{2}\log_7(2x-3)$

4.7 Exponential Eqns

Remember: Defn log

$x^a = b \Leftrightarrow \log_x b = a$

Exponential growth/decay models:

$$A = A_0 e^{rt}$$

r = growth/decay rate ($r > 0$ growth, $r < 0$ decay)

t = time

A_0 = initial amt of stuff

A = amt of stuff at time t

Strategy to Solve Exponential Eqns

① Isolate exponential term

② (i) use defn of log to rewrite it as log eqn

or (ii) take log of both sides (of same base as exponential)

③ finish solving

Ex 1 Solve

(a) $6^{5x-3} = 5$

(b) $3(5^x) + 30 = 105$

4.7 (cont)

Ex 2 Solve.

(a) $\left(1+\frac{0.055}{12}\right)^{12x}=2$

(b) $850=55(4)^{0.08x^2}$

(c) $850=55(10)^{0.08x^2}$

4.7 (cont)

Ex3 Solve.

(a) $\frac{4^x - 4^{-x}}{5} = 50$

(b) $x^2(3^x) = 9(3^x)$

4.7 (cont)

Ex 4 Solve for n. $N = 80(1 - e^{kn})$

Ex 5 The half-life of ^{22}Na sodium-22 is 2.6 yrs. If 15.5 g of an original 100-g specimen remains, how many years have elapsed?