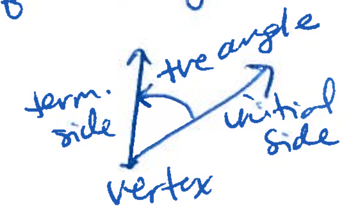
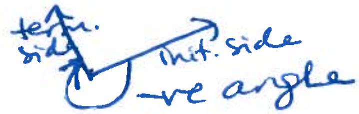
5.1 Angles

Vocab

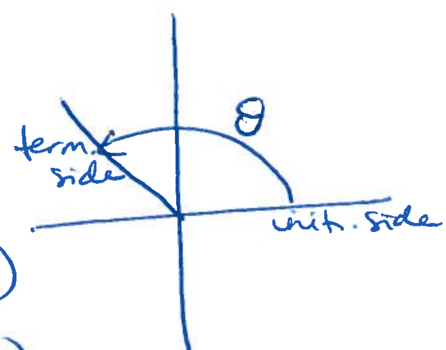
angle: formed by rotating a ray about its endpt (called a **vertex**) from some initial position (called **initial side**) to some terminal position (called **terminal side**). Measure of an angle tells how much rotation there is.



positive angle: counterclockwise
negative angle: clockwise



standard position: vertex at origin in Cartesian coordinate system; initial side on positive x-axis.



one revolution around circle = $360°$ (360 degrees)

$1° = 60'$ (1 degree = 60 minutes)

$1' = 60''$ (1 minute = 60 seconds)

$360° = 2\pi$ radians

☆ always use radians when you need a unit-less measure.

$\Rightarrow \dfrac{360°}{2\pi} = \dfrac{180°}{\pi}$ is a form of one

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coterminal angles: angles that both start on the x-axis and have same terminal side, even though they differ in how we write them down

5.1 Angles (cont)

Arc length: cut by a central angle (measured in radians) from a circle of radius r is denoted by s and $s = r\theta$ (or $\frac{s}{r} = \theta$)

(this is also actually how radians can be defined)

Ex 1 Convert + sketch

(a) 60° (b) -240° (c) 135°

(d) $\pi/4$ (e) $-2\pi/3$ (f) $3\pi/2$

5.1 (cont)

Ex 2 sketch and name reference angle (in radians)

(a) -225°

(b) 85°

(c) $\frac{11\pi}{6}$

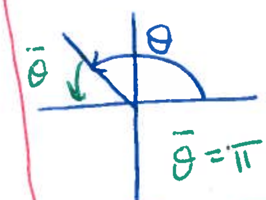
Reference Angles

$\bar{\theta}$ is reference angle for θ and it's the acute angle whose terminal side is terminal side of θ + other side of angle is closest x-axis.

$\theta \in Q1$

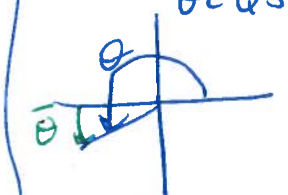
$\bar{\theta} = \theta$

$\theta \in Q2$



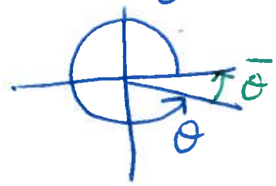
$\bar{\theta} = \pi - \theta$

$\theta \in Q3$



$\bar{\theta} = \theta - \pi$

$\theta \in Q4$



$\bar{\theta} = 2\pi - \theta$

note:
$\bar{\theta}$ is always +ve and $0^\circ < \bar{\theta} < 90^\circ$

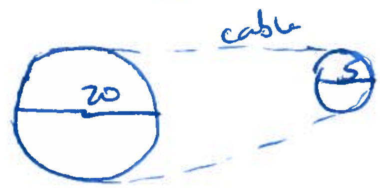
(d) $5\pi/3$

5.1 (cont)

Ex 3 Find the length of the arc if central angle $\theta = 2\pi/3$ and radius of circle is $r = 15$cm.

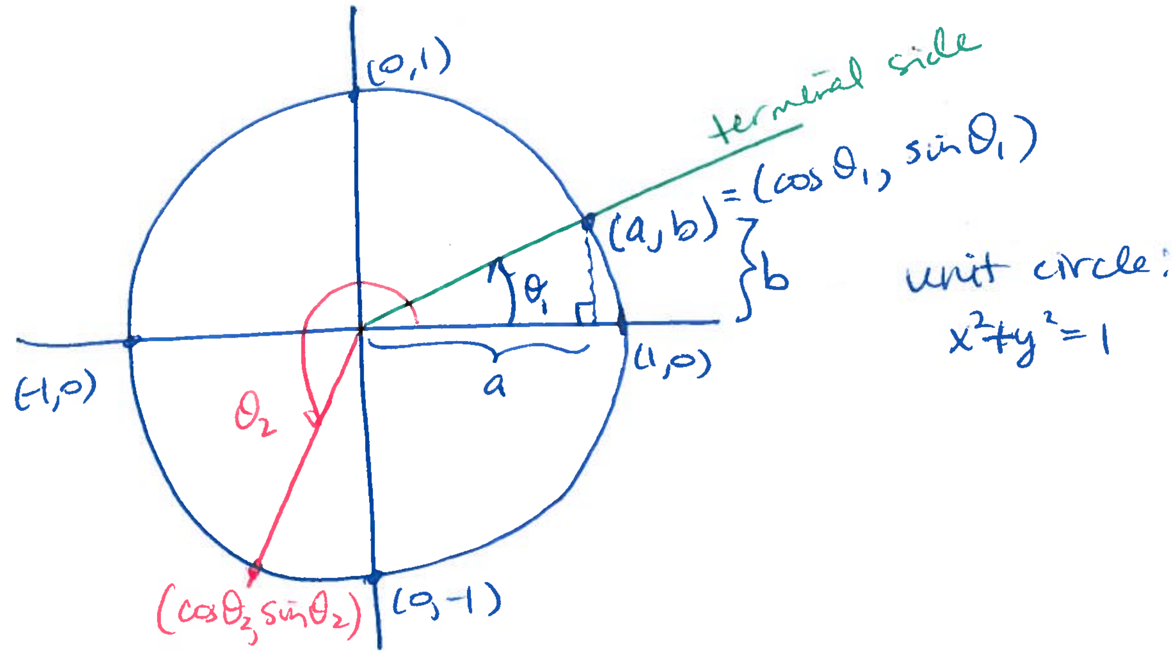
Ex 4 Through how many revolutions does a pulley w/ 5 cm diameter turn when 1 m of cable is pulled through it w/o slippage?

(1 m = 100 cm)



5.2 Fundamentals (of Trigonometry)

This gives defns of trigonometric fns on a unit circle (a circle centered at (0,0) w/ radius of 1).



Defn

$a = \cos\theta$ $\qquad$ $b = \sin\theta$

So every pt on the unit circle has coords $(\cos\theta, \sin\theta)$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{b}{a}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \frac{1}{\tan\theta} = \frac{a}{b}$$

$$\csc\theta = \frac{1}{\sin\theta} = \frac{1}{b}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{a}$$

5.2 (cont)

Ex 1 Use the circle (on grid) below to answer these questions.

(a) $\cos 250°$

$\sin 250°$

(b) $\sin(-45°)$

(c) $\tan(-6)$

(d) $\cot(5)$

(e) $\csc(-3)$

Pythagorean Identities

① $\sin^2\theta + \cos^2\theta = 1$

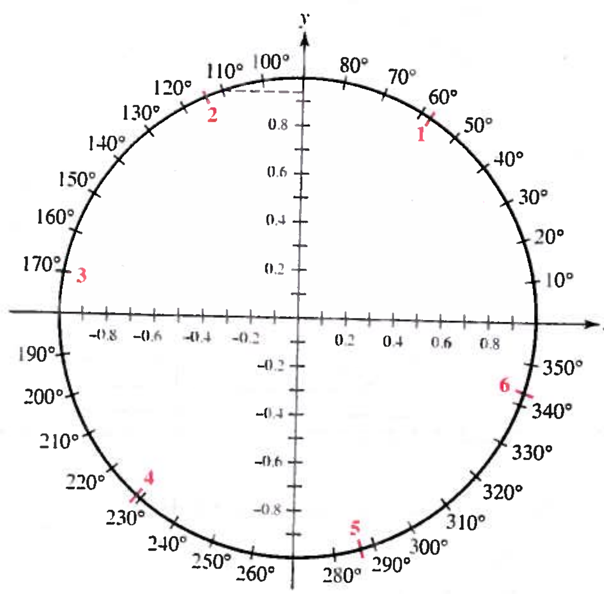
(this comes from unit circle defns)

② $\dfrac{\sin^2\theta}{\sin^2\theta} + \dfrac{\cos^2\theta}{\sin^2\theta} = \dfrac{1}{\sin^2\theta}$

$\Leftrightarrow \quad 1 + \cot^2\theta = \csc^2\theta$

③ $\dfrac{\sin^2\theta}{\cos^2\theta} + \dfrac{\cos^2\theta}{\cos^2\theta} = \dfrac{1}{\cos^2\theta}$

$\Leftrightarrow \quad \tan^2\theta + 1 = \sec^2\theta$



Ex 2 Specify which quadrant(s) θ lies in, given the specified information.

(a) $\sin\theta = -0.85$, $\cos\theta > 0$

(b) $\cot\theta > 0$

(c) $\cos\theta < 0$ and $\tan\theta > 0$

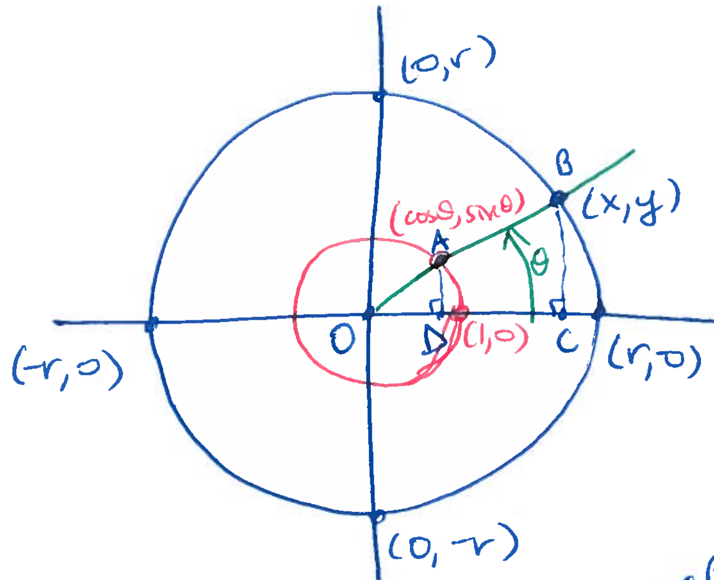
(d) $\csc\theta = -1.35$

5.2 (cont)

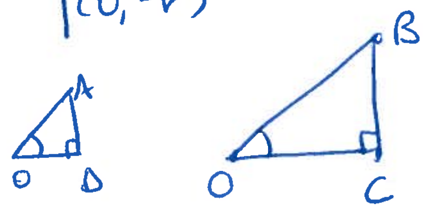
Ex3 Given $\cos\theta = \frac{-5}{13}$ and $\tan\theta > 0$, find all other trig fn values of θ.

5.3 Trigonometric Fns of Any Angle

Expand our defn of trig fns to a circle of any size, centered at origin w/ radius r.



By similar Δs.



$$\frac{AD}{OD} = \frac{BC}{OC}, \quad \frac{AD}{OA} = \frac{BC}{OB}, \quad \text{etc.}$$

but $\dfrac{AD}{OA} = \dfrac{\sin\theta}{1} = \dfrac{BC}{OB} = \dfrac{y}{r} \Rightarrow y = r\sin\theta$

and $\dfrac{OD}{OA} = \dfrac{\cos\theta}{1} = \dfrac{OC}{OB} = \dfrac{x}{r} \Rightarrow x = r\cos\theta$

$\cos\theta = \dfrac{x}{r}$, $\sin\theta = \dfrac{y}{r}$, $\tan\theta = \dfrac{y}{x}$ $(x \neq 0)$

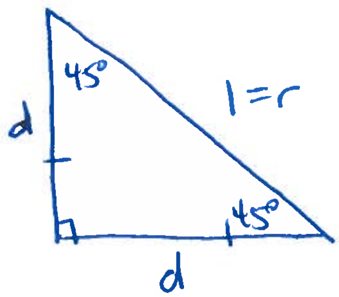
$\cot\theta = \dfrac{x}{y}$ $(y \neq 0)$, $\csc\theta = \dfrac{r}{y}$ $(y \neq 0)$, $\sec\theta = \dfrac{r}{x}$ $(x \neq 0)$

where (x,y) is any pt

5.3 (cont)

Common Angles

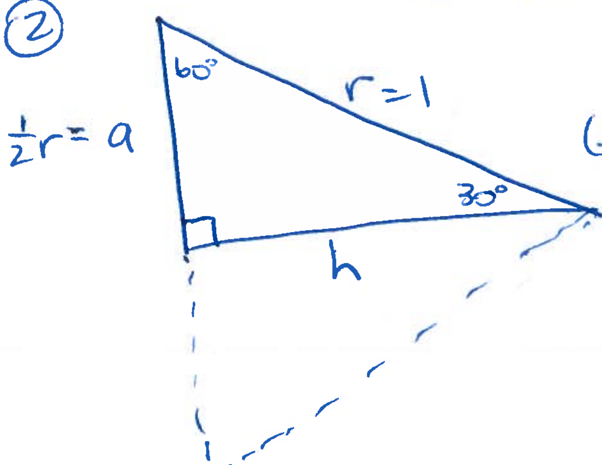
(1)



(isosceles right Δ)
(assume r given, as unit circle radius)

$30°-60°-90°$ Δ.

(2)



(assume r given)
(think of this as half an equilateral Δ)

5.3 (cont)

★ memorize table of exact values (pg 308)

Ex1 Find exact values.

(a) $\csc 30^\circ$

(b) $\csc \pi$

(c) $\tan 90^\circ$

(d) $\sin 0$

(e) $\cos 0$

(f) $\cot(\pi/4)$

(g) $\sin\left(\frac{17\pi}{4}\right)$

(h) $\cos\left(\frac{9\pi}{2}\right)$

5.3 (cont)

EX2 Find six trig fn values for angle whose terminal side goes through pt $(-6, 1)$.

EX3 Give exact values.

(a) $\sin^2\left(\frac{\pi}{6}\right) + \cos^2\left(\frac{\pi}{3}\right)$

(b) $\cos\left(\frac{\pi}{4} - \frac{\pi}{2}\right)$

(c) $\dfrac{2\tan 60^\circ}{1-\tan^2 60^\circ}$

(d) $\cos\left(\frac{\pi}{4}\right) - \cos\left(\frac{\pi}{2}\right)$