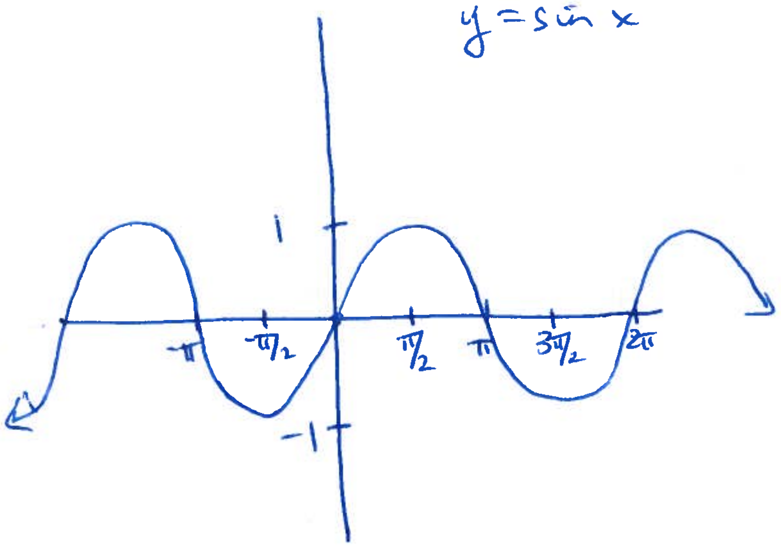
5.4 Graphs of Trigonometric Fns

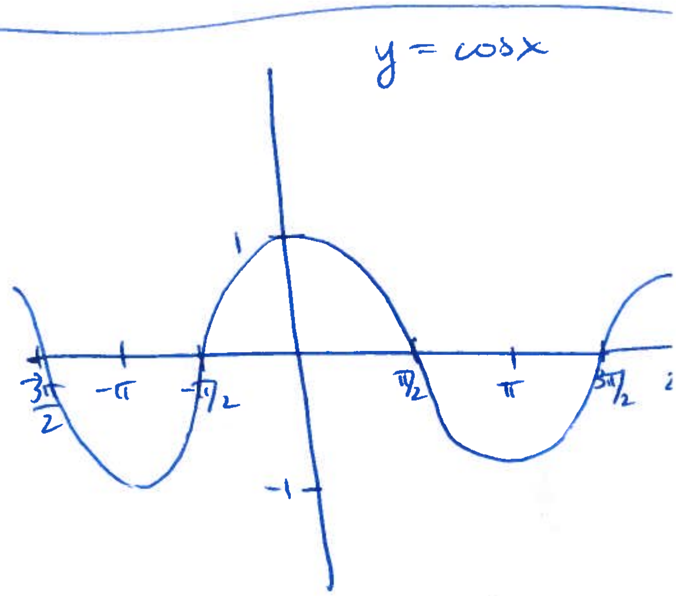
Vocab

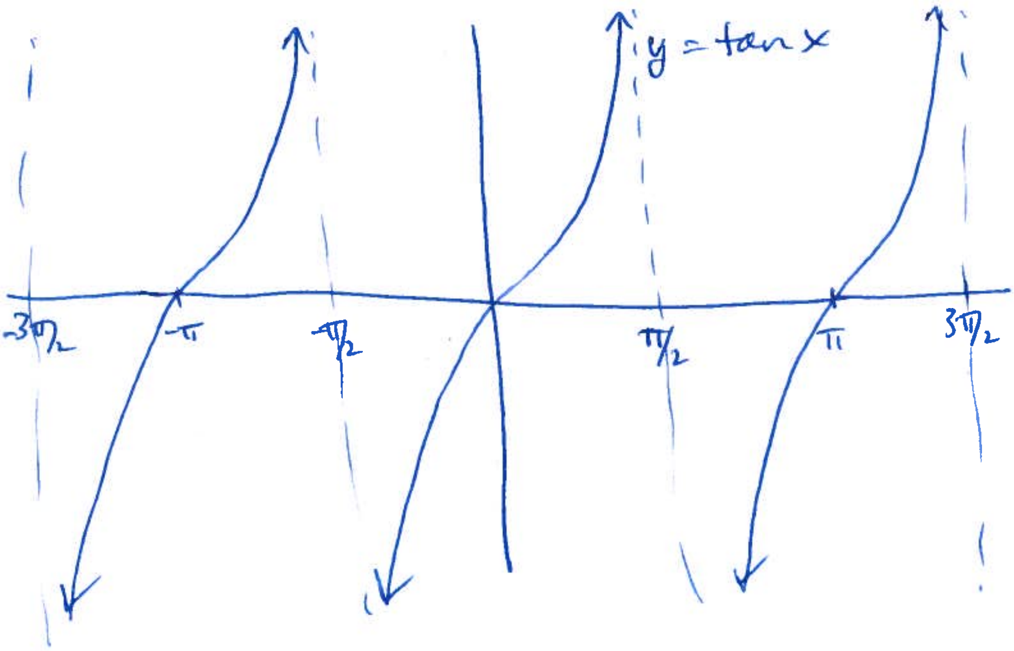
amplitude: half the distance from max to min heights

period: horizontal length of a cycle, i.e. length it goes before graph repeats itself.

frequency = $\frac{1}{\text{period}}$ = how many cycles in a given unit of time







5.4 (cont)

Ex 1 graph $y = \cot x$

Ex 2 graph $y = \csc x$ and $y = \sec x$

5.4 (cont)

Ex 3 Graph these fns. Give period & amplitude.

(a) $y = \sin(x - \pi/2) + 2$

(b) $y = 3\cos(x + \pi/3) - 1$

(c) $y = 2\sin(3x + \pi)$

5.5 Inverse Trigonometric Fns

WARNING: In order to define inverse fns for trigonometric fns (since none of them pass the horizontal line test), we MUST restrict the domain! This means, as a consequence, the inverse fn does not always undo the trig fn.

① $y=\sin^{-1}x \iff x=\sin y$ and $-\pi/2 \le y \le \pi/2$

② $y=\cos^{-1}x \iff x=\cos y$ and $y\in[0,\pi]$

③ $y=\tan^{-1}x \iff x=\tan y$ and $y\in(-\pi/2, \pi/2)$

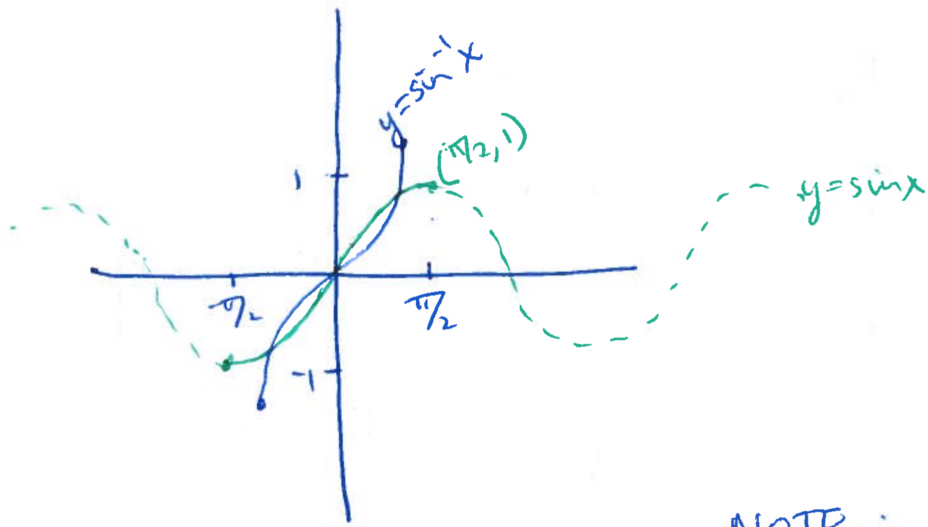
look at nice tables of graphs on pgs 336-337

④ $y=\cot^{-1}x \iff x=\cot y$ and $y\in(0,\pi)$

⑤ $y=\sec^{-1}x \iff x=\sec y$ and $y\in[0,\pi]$, $y\neq\pi/2$

⑥ $y=\csc^{-1}x \iff x=\csc y$ and $y\in[-\pi/2,\pi/2]$, $y\neq 0$

ex $y=\sin^{-1}x$



NOTE: another way to write $\sin^{-1}x$ fn is arcsin x (and same for all the others)

5.5 (cont)

Ex 1 Find these values (w/o a calculator).

(a) $\sin^{-1}(1)$

(b) $\cos^{-1}(1)$

(c) $\sin^{-1}\left(\frac{1}{2}\right)$

(d) $\tan^{-1}\left(\frac{\sqrt{3}}{3}\right)$

(e) $\text{arccot}(-\sqrt{3})$

(f) $\arcsin\left(\frac{\sqrt{3}}{2}\right)$

(g) $\arctan(0)$

(h) $\tan^{-1}(4)$

5.5 (cont)

Ex 2 Evaluate (exactly, w/o a calculator).

(a) $\cos(\tan^{-1}\sqrt{3})$

Inverse Identities

(i) $\cot^{-1}x = \begin{cases} \tan^{-1}(\frac{1}{x}), & x > \\ \pi/2, & x = 0 \\ \pi + \tan^{-1}(\frac{1}{x}), & x \end{cases}$

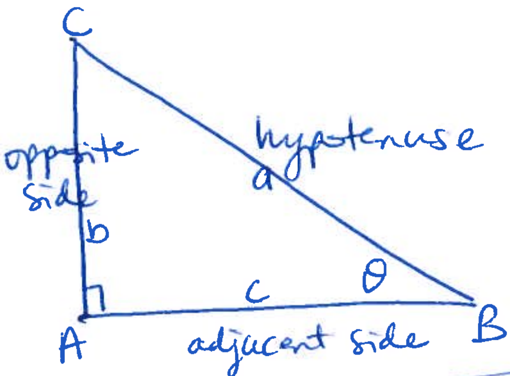
(ii) $\sec^{-1}x = \cos^{-1}(\frac{1}{x})$, $x \geq 1$ or $x \leq -1$

(iii) $\csc^{-1}x = \sin^{-1}(\frac{1}{x})$, $x \geq 1$ or $x \leq -1$

(b) $\sin(\cos^{-1}(\frac{2}{3}))$

(c) $\tan(\sec^{-1}x)$

5.6 Right Triangles



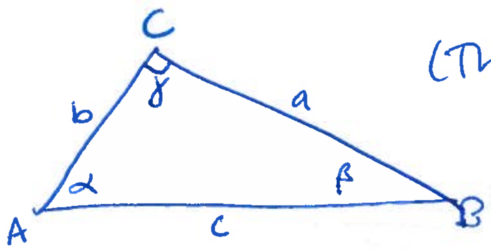
by Pythagorean Thm, we know $b^2+c^2=a^2$ in this Δ.

But we can also define trig fns here.

$$\sin\theta = \frac{opp}{hyp}, \quad \cos\theta = \frac{adj}{hyp}, \quad \tan\theta = \frac{opp}{adj}$$

only in a <u>right</u> Δ

Ex 1



(This is the default way your book will label right Δ.)

(a) If $a=145$ & $b=240$, find c, α, β.

(b) If $a=49$ and $\alpha=45°$, find b, c, β.

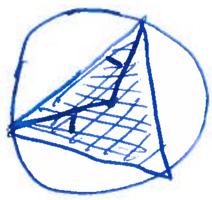
5.6 (cont)

Ex2 Using the reference Δ for $u = a\sec\theta$, write all six trig fns (of θ).

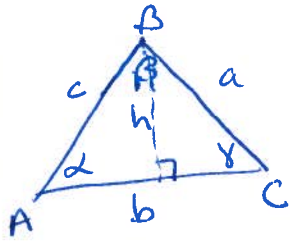
Ex3 The angle of elevation of the top of a pyramid from a pt on the ground 351 ft from a pt directly below the top is 52°. Find the height of the pyramid.

5.6 (cont)

Ex 4 Find area of equilateral Δ inscribed in a unit circle



Area of Triangle



area $= \frac{1}{2}bh$

but $\sin\alpha = \frac{h}{c}$

$\Rightarrow h = c\sin\alpha$

$\Rightarrow$ $\boxed{\text{area} = \frac{1}{2}bc\sin\alpha}$

or $\sin\gamma = \frac{h}{a}$

$\Rightarrow h = a\sin\gamma$

$\Rightarrow$ $\boxed{\text{area} = \frac{1}{2}ba\sin\gamma}$

or $\boxed{\text{area} = \frac{1}{2}ac\sin\beta}$