6.2 Proving Identities

Ex 1 Prove $\tan\theta + \cot\theta = \sec\theta\csc\theta$

Ex 2 Prove $1+\sin^2\theta = 2-\cos^2\theta$

* An identity is an eqn that's true for all values of the variable.

Strategy

(1) Start w/ LHS or RHS (LHS = left hand side)

(2) Do legal algebra steps to that expression until it looks like the other side.

WARNING: Do NOT "do the same thing to both sides", because you can't assume identity is true before you prove it to be true.

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6.2 (cont)

Ex 3 Prove $\dfrac{1+\cos(2\lambda)\sec(2\lambda)}{\tan(2\lambda)+\sec(2\lambda)}$

Ex 4 Prove $\dfrac{2\tan^2\theta + 2\tan\theta\sec\theta}{\tan\theta+\sec\theta-1} = \tan\theta+\sec\theta+1$

6.2 (cont)

Ex 5 Prove or disprove: $\cos(\alpha-\beta)=\cos\alpha-\cos\beta$.

Ex 6 prove $\dfrac{1-\cos\theta}{1+\cos\theta}=\left(\dfrac{1-\cos\theta}{\sin\theta}\right)^2$

6.3 Addition Laws

Addition Identities

(✱) $\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\tan(\alpha-\beta) = \frac{\tan\alpha - \tan\beta}{1+\tan\alpha\tan\beta}$$

Cofunction Identities

(a) $\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$

(b) $\sin\left(\frac{\pi}{2}-\theta\right) = \cos\theta$

(c) $\tan\left(\frac{\pi}{2}-\theta\right) = \cot\theta$

Opposite-Angle Identities

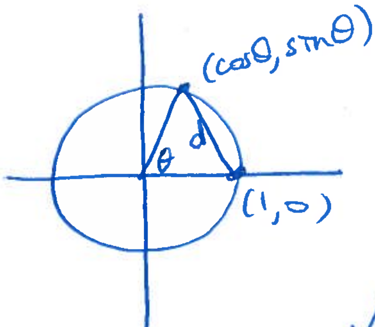
$\cos(-\theta) = \cos\theta$

$\sin(-\theta) = -\sin\theta$

$\tan(-\theta) = -\tan\theta$

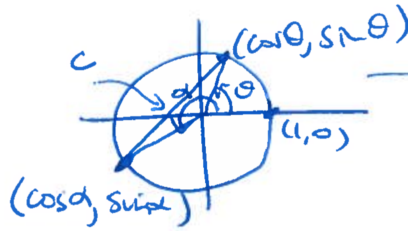
i.e. $\sin\theta$ & $\tan\theta$ are odd fns, $\cos\theta$ is even

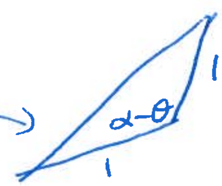
Proof of (✱)



$$d = \sqrt{(\cos\theta-1)^2 + \sin^2\theta}$$

$\Rightarrow$





$$\Rightarrow c = \sqrt{(\cos(\alpha-\theta)-1)^2 + \sin^2(\alpha-\theta)}$$

$$= \sqrt{\cos^2(\alpha-\theta) - 2\cos(\alpha-\theta) + 1 + \sin^2(\alpha-\theta)}$$

$$= \sqrt{2 - 2\cos(\alpha-\theta)}$$

but c also is distance from $(\cos\theta, \sin\theta)$ to $(\cos\alpha, \sin\alpha)$

$$\Rightarrow c = \sqrt{(\cos\theta - \cos\alpha)^2 + (\sin\theta - \sin\alpha)^2}$$

6.3 (cont)

$$\Rightarrow c = \sqrt{\cos^2\theta - 2\cos\theta\cos\alpha + \cos^2\alpha + \sin^2\theta - 2\sin\theta\sin\alpha + \sin^2\alpha}$$

$$c = \sqrt{2 - 2\cos\theta\cos\alpha - 2\sin\theta\sin\alpha}$$

$$\Rightarrow \sqrt{2-2\cos\theta\cos\alpha - 2\sin\theta\sin\alpha} = \sqrt{2 - 2\cos(\alpha-\theta)}$$

$$\Rightarrow \cos\theta\cos\alpha - \sin\theta\sin\alpha = \cos(\alpha-\theta)$$

Ex 1 Change to fn of θ only.

(a) $\cos\left(\frac{\pi}{3} - \theta\right)$

(b) $\tan\left(\frac{\pi}{4} + \theta\right)$

6.3 (cont)

Ex 2 Evaluate $\sin 18^\circ \cos 23^\circ + \cos 18^\circ \sin 23^\circ$

Ex 3 Find exact value of $\sin(195^\circ)$, $\cos(195^\circ)$ & $\tan(195^\circ)$

Ex 4 Evaluate $\sin\left(\cos^{-1}\left(\frac{2}{3}\right) + \sin^{-1}\left(\frac{1}{2}\right)\right)$

6.3 (cont)

Ex 5 Prove $\sin(\alpha+\beta)\cos\beta - \cos(\alpha+\beta)\sin\beta = \sin\alpha$.

6.4 Miscellaneous Identities

Double Angle Identities

$\cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\sin(2\theta) = 2\sin\theta\cos\theta$

$\tan(2\theta) = \dfrac{2\tan\theta}{1-\tan^2\theta}$

Half-Angle Identities

$\cos\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1+\cos\theta}{2}}$

$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\dfrac{1-\cos\theta}{2}}$

$\tan\left(\frac{\theta}{2}\right) = \dfrac{\sin\theta}{1+\cos\theta}$

Product-to-Sum Identities

$2\cos\alpha\cos\beta = \cos(\alpha-\beta) + \cos(\alpha+\beta)$

$2\sin\alpha\sin\beta = \cos(\alpha-\beta) - \cos(\alpha+\beta)$

$2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)$

$2\cos\alpha\sin\beta = \sin(\alpha+\beta) - \sin(\alpha-\beta)$

Sum-to-Product Identities

$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$

$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$

$\sin x - \sin y = 2\sin\left(\frac{x-y}{2}\right)\cos\left(\frac{x+y}{2}\right)$

Ex 1 If $\theta \in [0, 2\pi)$, give possible quadrants for these angles.

(a) $\frac{\theta}{2}$ if $\theta \in Q2$

(b) 2θ if $\theta \in Q2$

(c) $\frac{\theta}{2}$ if $\cos\left(\frac{\theta}{2}\right) = -\sqrt{\dfrac{1+\cos\theta}{2}}$

(d) 2θ if $\theta \in Q3$ and $\cos(2\theta) = -5/9$

6.4 (cont)

Ex 2 Evaluate.

$$-\sqrt{\frac{1-\cos(420^\circ)}{2}}$$

Ex 3 Write as sum or difference.

$$2\cos 70^\circ \sin 24^\circ$$

Ex 4 Write as product.

$$\sin(3x) - \sin(2x)$$

6.4 (cont)

Ex5 Solve $\sin(2x) - \cos x = 0$

Ex6 Find $\sin(2\theta)$, $\cos(2\theta)$, $\tan(2\theta)$ and $\sin\left(\frac{\theta}{2}\right)$, $\cos\left(\frac{\theta}{2}\right)$, $\tan\left(\frac{\theta}{2}\right)$

if $\tan\theta = \frac{-3}{4}$, $\theta \in Q2$.