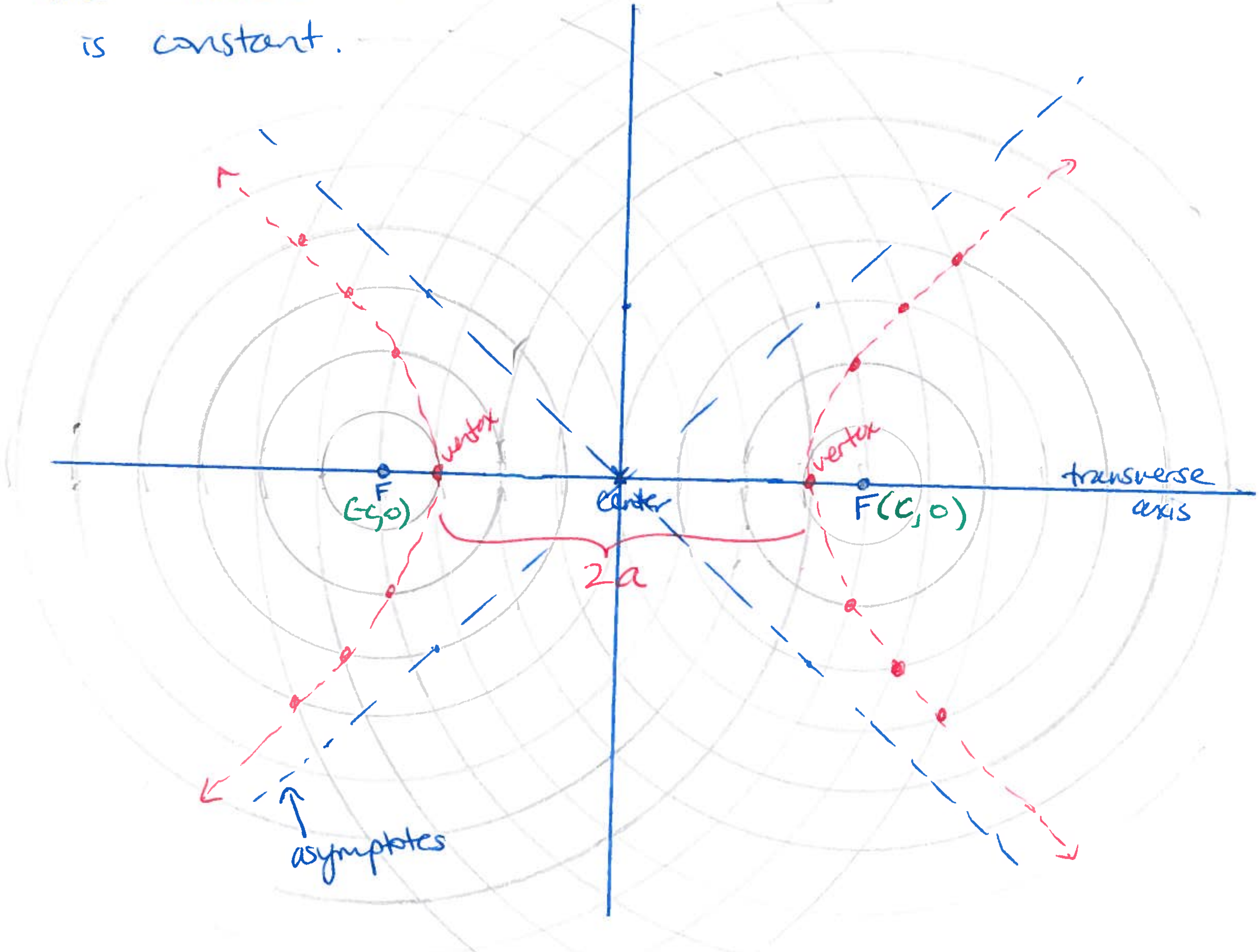
7.3 Hyperbolas

hyperbola: set of all points in a plane such that the difference of its distances from two fixed pts (called foci) is constant.



eqn of this hyperbola

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$$

foci at $(\pm c, 0)$

vertices at $(\pm a, 0)$

let $b^2 = c^2 - a^2$

From generic conic section form: hyperbola is

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

w/ A and C have different signs

7.3 (cont)

Eqn of Hyperbola

distance from (x,y) on hyperbola to $(-c,0)$ and $(c,0)$ has constant difference of $2a$

① ②

$$\Rightarrow \left|\sqrt{(x+c)^2+(y-0)^2} - \sqrt{(x-c)^2+(y-0)^2}\right| = 2a$$

Case 1: ① ≥ ② ⇒ abs. value signs do nothing

$$\sqrt{(x+c)^2+y^2} - \sqrt{(x-c)^2+y^2} = 2a$$

$$\left(\sqrt{(x+c)^2+y^2}\right)^2 = \left(2a+\sqrt{(x-c)^2+y^2}\right)^2$$

$$(x+c)^2+\cancel{y^2} = 4a^2 + 4a\sqrt{(x-c)^2+y^2} + (x-c)^2 + \cancel{y^2}$$

$$\cancel{x^2}+2cx+\cancel{c^2} = 4a^2 + 4a\sqrt{(x-c)^2+y^2} + \cancel{x^2} - 2cx + \cancel{c^2}$$

$$\frac{4cx-4a^2}{4} = a\sqrt{(x-c)^2+y^2} \iff (cx-a^2)^2 = \left(a\sqrt{(x-c)^2+y^2}\right)^2$$

$$c^2x^2-2cxa^2+a^4 = a^2\left((x-c)^2+y^2\right)$$

$$c^2x^2-\cancel{2cxa^2}+a^4 = a^2x^2-\cancel{2cxa^2}+c^2a^2+a^2y^2$$

$$x^2(c^2-a^2) - a^2y^2 = c^2a^2 - a^4$$

$$x^2(c^2-a^2) - a^2y^2 = a^2(c^2-a^2)$$

$$\frac{x^2\cancel{(c^2-a^2)}}{a^2\cancel{(c^2-a^2)}} - \frac{\cancel{a^2}y^2}{\cancel{a^2}(c^2-a^2)} = 1$$

$$\frac{x^2}{a^2} - \frac{y^2}{c^2-a^2} = 1$$

let $b^2 = c^2 - a^2$

$$\Rightarrow \boxed{\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1}$$

7.3 (cont)

Ex 1 Graph

(a) $\frac{x^2}{4} - \frac{y^2}{9} = 1$

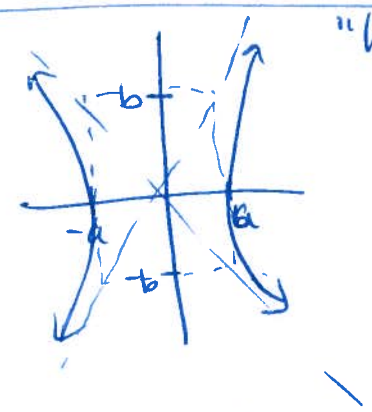
(b) $4(y+1)^2 - 16(x-2)^2 = 1$

Standard Hyperbola w/ center (h,k)

$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$ (horizontal)
vertices $(h \pm a, k)$

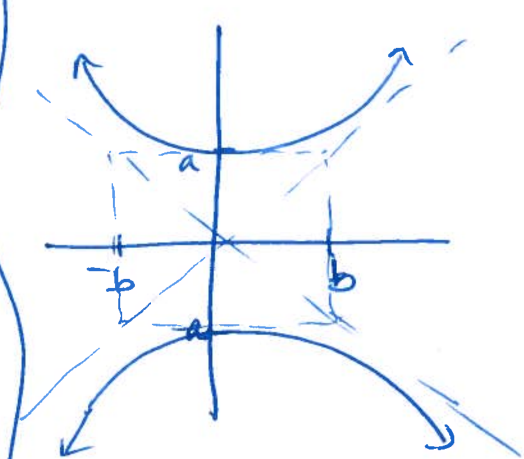
$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$ (vertical)
vertices $(h, k \pm a)$

"horizontal"



foci: $(\pm c, 0)$
vertices: $(\pm a, 0)$
center: $(0,0)$
eqn: $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$

"vertical"



foci: $(0, \pm c)$
vertices: $(0, \pm a)$
center: $(0,0)$
eqn: $\frac{y^2}{a^2} - \frac{x^2}{b^2} = 1$

7.3 (cont)

Ex2 Identify each conic and graph.

(a) $5(x-3)^2 - (y+2)^2 = 15$

(b) $\dfrac{x-3}{9} + \dfrac{y-2}{25} = 1$

Given

$Ax^2 + Cy^2 + Dx + Ey + F = 0$

(i) does $A=0$ **and** $C=0$?
then it's a line

(ii) does $A=0$ **or** $C=0$ (but not both zero)?
then it's a parabola

(iii) do A and C both have same sign? (not zero)
then it's an ellipse.
(if $A=C$, it's a circle)

(iv) do A and C have opposite signs?
then it's a hyperbola

★ See pg 460 in book for a great recap summary of conic sections!!

7.3 (cont)

Ex 3 Identify and graph.

(a) $9(x+3)^2 + 4(y-2)^2 = 0$

(b) $9(x+3)^2 + 4(y-2) = 0$

(c) $9x^2 + 25y^2 - 54x - 200y + 256 = 0$

7.3 (cont)

eccentricity of hyperbola

$\boxed{\varepsilon = \frac{c}{a}}$ for hyperbola $c > a$

$\Rightarrow \varepsilon > 1$

(for parabola $\varepsilon = 1$, for ellipse $a > c \Rightarrow \varepsilon < 1$)

Ex 4 Find eqn for conic w/ foci at $(-1, 3)$ and $(7, 3)$ w/ $\varepsilon = 3/2$.

8.1 Sequences

a sequence can be thought of as a discrete function, i.e. a fn that only takes natural #s as inputs.

We'll look at 3 types of sequences

Arithmetic Sequence	Geometric sequence	Fibonacci-type Sequence
• the next term in this sequence is the last term + d (d fixed, called the common difference)	• the next term is the last term multiplied by a fixed # r (called the common ratio)	• next term is the sum of previous two terms (w/ very first two terms given)
• recursive formula: a_1 is given, $a_n = a_{n-1} + d$	• recursive formula g_1 given $g_n = g_{n-1} \cdot r$	• recursive formula $\boxed{S_1, S_2 \text{ given} \quad S_n = S_{n-1} + S_{n-2}}$
• iterative formula: $\boxed{a_n = a_1 + (n-1)d \quad n = 1, 2, 3, \ldots}$	• iterative formula $\boxed{g_n = g_1 r^{n-1} \quad n = 1, 2, 3, \ldots}$	• iterative formula DNE
• examples in life: simple interest	• examples in life Compound interest population growth	• examples in life predicting population and used in lots of sciences * (see book pg 503-506 for great write-up)

note: recursive formula depends on previous value(s) in sequence

iterative (or direct) formula only depends on n (the counter or input)

8.1 (cont)

Ex1 Classify sequence; state pattern; supply next term.

(a) $2, 4, 8, 16, \ldots$

(b) $5, 15, 25, \ldots$

(c) $8, 6, 7, 5, 6, 4, \ldots$

8.1 (cont)

Ex 2 Classify sequence as arithmetic or geometric or neither.

Find first 3 terms.

(a) $s_n = 7 - 3n$

(b) $s_n = \frac{10}{2^{n-1}}$

(c) $\{(-1)^n(n+1)\}$

(d) $\{\frac{2}{3}\}$

8.1 (cont)

Ex 3 Find 6^{th} term of $S_n = \dfrac{(-1)^{n+1}\, 5^{n+1}}{2}$

Ex 4 Find first five terms of

$S_1 = 3, \quad S_n = \frac{1}{3} S_{n-1}, \quad n \geq 2.$