8.2 Limit of a Sequence

Defn $\{s_n\}$ converges to L (a fixed #), $L = \lim_{n\to\infty} s_n$, if terms of $\{s_n\}$ can be made as close to L as possible when n is sufficiently large.
If a sequence does not converge, then it diverges.

Properties If $\lim_{n\to\infty} s_n = L$ & $\lim_{n\to\infty} t_n = M$, then $\begin{pmatrix} L<\infty \\ M<\infty \end{pmatrix}$

① $\lim_{n\to\infty} (as_n + bt_n) = aL + aM$

② $\lim_{n\to\infty} (s_n t_n) = LM$

③ $\lim_{n\to\infty} \frac{s_n}{t_n} = \frac{L}{M}$ (if $M \neq 0$)

④ $\lim_{n\to\infty} \sqrt[m]{s_n} = \sqrt[m]{L}$ (provided $\sqrt[m]{s_n}$ is well defined)

Defn $\lim_{n\to\infty} s_n = \infty$ means $\forall m>0$, $s_n > m$ for all sufficiently large n.

and $\lim_{n\to\infty} s_n = -\infty$ means $\forall M$, $s_n < M$ for sufficiently large n

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★ If f is a continuous fn, such that $s_n = f(n)$ $\forall$ $n=1,2,3,\ldots$ (i.e. s_n is the discrete counterpart to f) and $\lim_{x\to\infty} f(x) = L$, then $\lim_{n\to\infty} s_n = L$ also.

Ex1 compute limit of $\left\{ \frac{5n-1}{n+3} \right\}$

8.2 (cont)

Ex 2 Compute limits, if they exist.

(a) $\left\{ \left(\frac{-1}{3}\right)^n \right\}$

(b) $\left\{ \frac{8n^2 - 7n}{4n^3 + 5n + 1} \right\}$

(c) $\{ \ln(3n) - \ln(2n+1) \}$

8.2 (cont)

Ex 3 Compute limits, if they exist.

(a) $\left\{\left(1-\frac{5}{n}\right)^n\right\}$

(b) $\left\{\frac{\ln n}{n^2}\right\}$

8.3 Series

a series is the sum of terms in a sequence.

$S_n = a_1 + a_2 + \dots + a_n$ where $\{a_n\}$ is the sequence.

then S_n is the n^{th} sum (a finite series)

notation: $S_n = a_1 + a_2 + \dots + a_n = \sum_{i=1}^{n} a_i$

↑summation sign

Ex1 Evaluate $\sum_{k=1}^{4} (-1)^k (k^2+1)$

Ex2 Given sequence $xyz, xy, \frac{xy}{z}, \dots,$ find general n^{th} term of sequence and write S_n for this sequence.

8.3 (cont)

Ex 3 Find the sums

(a) $17+9+1+\cdots+(-55)$

(b) $-12-36-108-\cdots-2916$

Sum of Arithmetic sequence

$A_n = \sum_{k=1}^{n} a_k = n\left(\frac{a_1+a_2}{2}\right)$

for arithmetic sequence $a_1, a_2, \ldots, a_n$

Pf

Sum of geometric sequence (finite) $\{g_n\}$

$g_n = g_1 r^{n-1}$

$G_n = \sum_{k=1}^{n} g_k = \frac{g_1(1-r^n)}{1-r}$

Pf

8.3 (cont)

Ex 4 Find infinite sum.

$-20+10-5+\ldots$

Sum of infinite Geometric Series

$$G=\lim_{n\to\infty}\sum_{k=1}^{n} g_k$$

$$=\lim_{n\to\infty}\frac{g_1(1-r^n)}{1-r}$$

$$=\begin{cases}\infty & \text{if } |r|\ge 1\\ \dfrac{g_1}{1-r} & \text{if } |r|<1\end{cases}$$

Ex 5 A tennis ball is dropped from a height of 10 ft. If the ball rebounds $\frac{2}{3}$ of its height on each bounce, how far will the ball travel (vertically) before stopping?