3.6 Ex4 Solve.

$$x^4 - 12x^3 - 13 = 6(3 - 2x - 5x^2)$$

$$x^4 - 12x^3 - 13 = 18 - 12x - 30x^2$$

(★) $$x^4 - 12x^3 + 30x^2 + 12x - 31 = 0$$

Descartes Rule of Signs: ① 3 sign changes ⇒ 3 or 1 positive $\mathbb{R}$ roots

② $(-x)^4 - 12(-x)^3 + 30(-x)^2 + 12(-x) - 31 = x^4 + 12x^3 + 30x^2 - 12x - 31$

1 sign change ⇒ 1 negative $\mathbb{R}$ root

Possible Rational Roots

$\pm 1, \pm 31$

1	1	−12	30	12	−31
		1	−11	19	31
	1	−11	19	31	0
−1		−1	12	−31	
	1	−12	31	0	

1 is a zero ⇔ $(x-1)$ is a factor

−1 is a root ⇔ $(x+1)$ is a factor

⇒ we have (★) as $(x-1)(x+1)(x^2 - 12x + 31) = 0$

and we can finish off the rest w/ quadratic formula.

$$x^2 - 12x + 31 = 0 \Rightarrow x = \frac{12 \pm \sqrt{144 - 4(31)}}{2}$$

$$= \frac{12 \pm \sqrt{20}}{2} = \frac{2(6 \pm \sqrt{5})}{2} = 6 \pm \sqrt{5}$$

⇒ $x = 1, -1, 6+\sqrt{5}, 6-\sqrt{5}$ are all the solutions

3.6 Ex 6 $P(x) = (x^2+2x+5)(x^2-3x+5) = 0$

① ②

separately handle each quadratic factor

① $x^2+2x+5=0$

$x = \frac{-2 \pm \sqrt{4-4(5)}}{2}$

$x = \frac{-2 \pm \sqrt{-16}}{2}$

$x = \frac{-2 \pm 4i}{2}$

$x = -1 \pm 2i$

② $x^2-3x+5=0$

$x = \frac{3 \pm \sqrt{9-4(5)}}{2}$

$x = \frac{3 \pm \sqrt{-11}}{2}$

$x = \frac{3 \pm \sqrt{11}\,i}{2}$

$\Rightarrow$ sols are $x = -1+2i,\ -1-2i,\ \frac{3}{2}+\frac{\sqrt{11}}{2}i,\ \frac{3}{2}-\frac{\sqrt{11}}{2}i$